

NEUTRINO PHENOMENOLOGY

ESTEBAN ROULET*

*CONICET, Centro Atómico Bariloche
Av. Bustillo 9500, 8400, Bariloche, Argentina
E-mail: roulet@cab.cnea.gov.ar*

A general overview of neutrino physics is given, starting with a historical account of the development of our understanding of neutrinos and how they helped to unravel the structure of the Standard Model. We discuss why it is so important to establish if neutrinos are massive and the indications in favor of non-zero neutrino masses are discussed, including the recent results on atmospheric and solar neutrinos and their confirmation with artificial neutrino sources.

1. The neutrino story:

1.1. *The hypothetical particle:*

One may trace back the appearance of neutrinos in physics to the discovery of radioactivity by Becquerel one century ago. When the energy of the electrons (beta rays) emitted in a radioactive decay was measured by Chadwick in 1914, it turned out to his surprise to be continuously distributed. This was not to be expected if the underlying process in the beta decay was the transmutation of an element X into another one X' with the emission of an electron, i.e. $X \rightarrow X' + e$, since in that case the electron should be monochromatic. The situation was so puzzling that Bohr even suggested that the conservation of energy may not hold in the weak decays. Another serious problem with the 'nuclear models' of the time was the belief that nuclei consisted of protons and electrons, the only known particles by then. To explain the mass and the charge of a nucleus it was then necessary that it had A protons and $A - Z$ electrons in it. For instance, a ${}^4\text{He}$ nucleus would have 4 protons and 2 electrons. Notice that this total of six fermions would make the ${}^4\text{He}$ nucleus to be a boson, which is correct. However, the problem arose when this theory was applied for instance to ${}^{14}\text{N}$, since

*Work partially supported by Fundación Antorchas.

consisting of 14 protons and 7 electrons would make it a fermion, but the measured angular momentum of the nitrogen nucleus was $I = 1$.

The solution to these two puzzles was suggested by Pauli only in 1930, in a famous letter to the ‘Radioactive Ladies and Gentlemen’ gathered in a meeting in Tubingen, where he wrote: ‘I have hit upon a desperate remedy to save the exchange theorem of statistics and the law of conservation of energy. Namely, the possibility that there could exist in nuclei electrically neutral particles, that I wish to call neutrons, which have spin $1/2 \dots$ ’. These had to be not heavier than electrons and interacting not more strongly than gamma rays.

With this new paradigm, the nitrogen nucleus became $^{14}\text{N} = 14p + 7e + 7'n$, which is a boson, and a beta decay now involved the emission of two particles $X \rightarrow X' + e + 'n'$, and hence the electron spectrum was continuous. Notice that no particles were created in a weak decay, both the electron and Pauli’s neutron ‘ n ’ were already present in the nucleus of the element X , and they just came out in the decay. However, in 1932 Chadwick discovered the real ‘neutron’, with a mass similar to that of the proton and being the missing building block of the nuclei, so that a nitrogen nucleus finally became just $^{14}\text{N} = 7p + 7n$, which also had the correct bosonic statistics.

In order to account now for the beta spectrum of weak decays, Fermi called Pauli’s hypotetised particle the neutrino (small neutron), ν , and furthermore suggested that the fundamental process underlying beta decay was $n \rightarrow p + e + \nu$. He wrote ¹ the basic interaction by analogy with the interaction known at the time, the QED, i.e. as a vector \times vector current interaction:

$$H_F = G_F \int d^3x [\bar{\Psi}_p \gamma_\mu \Psi_n] [\bar{\Psi}_e \gamma^\mu \Psi_\nu] + h.c..$$

This interaction accounted for the continuous beta spectrum, and from the measured shape at the endpoint Fermi concluded that m_ν was consistent with zero and had to be small. The Fermi coupling G_F was estimated from the observed lifetimes of radioactive elements, and armed with this Hamiltonian Bethe and Peierls ² decided to compute the cross section for the inverse beta process, i.e. for $\bar{\nu} + p \rightarrow n + e^+$, which was the relevant reaction to attempt the direct detection of a neutrino. The result, $\sigma = 4(G_F^2/\pi)p_e E_e \simeq 2.3 \times 10^{-44} \text{cm}^2 (p_e E_e/m_e^2)$ was so tiny that they concluded ‘... This meant that one obviously would never be able to see a neutrino.’. For instance, if one computes the mean free path in water (with density $n \simeq 10^{23}/\text{cm}^3$) of a neutrino with energy $E_\nu = 2.5 \text{ MeV}$, typical of a

weak decay, the result is $\lambda \equiv 1/n\sigma \simeq 2.5 \times 10^{20}$ cm, which is 10^7 AU, i.e. comparable to the thickness of the Galactic disk.

It was only in 1956 that Reines and Cowan were able to prove that Bethe and Peierls had been too pessimistic, when they measured for the first time the interaction of a neutrino through the inverse beta process³. Their strategy was essentially that, if one needs 10^{20} cm of water to stop a neutrino, having 10^{20} neutrinos a cm would be enough to stop one neutrino. Since after the second war powerful reactors started to become available, and taking into account that in every fission of an uranium nucleus the neutron rich fragments beta decay producing typically 6 $\bar{\nu}$ and liberating ~ 200 MeV, it is easy to show that the (isotropic) neutrino flux at a reactor is

$$\frac{d\Phi_\nu}{d\Omega} \simeq \frac{2 \times 10^{20}}{4\pi} \left(\frac{\text{Power}}{\text{GWatt}} \right) \frac{\bar{\nu}}{\text{strad}}.$$

Hence, placing a few hundred liters of water (with some Cadmium in it) near a reactor they were able to see the production of positrons (through the two 511 keV γ produced in their annihilation with electrons) and neutrons (through the delayed γ from the neutron capture in Cd), with a rate consistent with the expectations from the weak interactions of the neutrinos.

1.2. *The vampire:*

Going back in time again to follow the evolution of the theory of weak interactions of neutrinos, in 1936 Gamow and Teller⁴ noticed that the $V \times V$ Hamiltonian of Fermi was probably too restrictive, and they suggested the generalization

$$H_{GT} = \sum_i G_i [\bar{p} O_i n] [\bar{e} O^i \nu] + h.c.,$$

involving the operators $O_i = 1, \gamma_\mu, \gamma_\mu \gamma_5, \gamma_5, \sigma_{\mu\nu}$, corresponding to scalar (S), vector (V), axial vector (A), pseudoscalar (P) and tensor (T) currents. However, since A and P only appeared here as $A \times A$ or $P \times P$, the interaction was parity conserving. The situation became unpleasant, since now there were five different coupling constants G_i to fit with experiments, but however this step was required since some observed nuclear transitions which were forbidden for the Fermi interaction became now allowed with its generalization (GT transitions).

The story became more involved when in 1956 Lee and Yang suggested that parity could be violated in weak interactions⁵. This could explain why the particles theta and tau had exactly the same mass and charge and only differed in that the first one was decaying to two pions while the second to three pions (e.g. to states with different parity). The explanation to the puzzle was that the Θ and τ were just the same particle, now known as the charged kaon, but the (weak) interaction leading to its decays violated parity.

Parity violation was confirmed the same year by Wu⁶, studying the direction of emission of the electrons emitted in the beta decay of polarized ^{60}Co . The decay rate is proportional to $1 + \alpha \vec{P} \cdot \hat{p}_e$. Since the Co polarization vector \vec{P} is an axial vector, while the unit vector along the electron momentum \hat{p}_e is a vector, their scalar product is a pseudoscalar and hence a non-vanishing coefficient α would imply parity violation. The result was that electrons preferred to be emitted in the direction opposite to \vec{P} , and the measured value $\alpha \simeq -0.7$ had then profound implications for the physics of weak interactions.

The generalization by Lee and Yang of the Gamow Teller Hamiltonian was

$$H_{LY} = \sum_i [\bar{p} O_i n] [\bar{e} O^i (G_i + G'_i \gamma_5) \nu] + h.c..$$

Now the presence of terms such as $V \times A$ or $P \times S$ allows for parity violation, but clearly the situation became even more unpleasant since there are now 10 couplings (G_i and G'_i) to determine, so that some order was really called for.

Soon the bright people in the field realized that there could be a simple explanation of why parity was violated in weak interactions, the only one involving neutrinos, and this had just to do with the nature of the neutrinos. Lee and Yang, Landau and Salam⁷ realized that, if the neutrino was massless, there was no need to have both neutrino chirality states in the theory, and hence the handedness of the neutrino could be the origin for the parity violation. To see this, consider the chiral projections of a fermion

$$\Psi_{L,R} \equiv \frac{1 \mp \gamma_5}{2} \Psi.$$

We note that in the relativistic limit these two projections describe left and right handed helicity states (where the helicity, i.e. the spin projection in the direction of motion, is a constant of motion for a free particle), but in general an helicity eigenstate is a mixture of the two chiralities. For a

massive particle, which has to move with a velocity smaller than the speed of light, it is always possible to make a boost to a system where the helicity is reversed, and hence the helicity is clearly not a Lorentz invariant while the chirality is (and hence has the desirable properties of a charge to which a gauge boson can be coupled). If we look now to the equation of motion for a Dirac particle as the one we are used to for the description of a charged massive particle such as an electron ($(i\partial - m)\Psi = 0$), in terms of the chiral projections this equation becomes

$$i\partial\Psi_L = m\Psi_R$$

$$i\partial\Psi_R = m\Psi_L$$

and hence clearly a mass term will mix the two chiralities. However, from these equations we see that for $m = 0$, as could be the case for the neutrinos, the two equations are decoupled, and one could write a consistent theory using only one of the two chiralities (which in this case would coincide with the helicity). If the Lee Yang Hamiltonian were just to depend on a single neutrino chirality, one would have then $G_i = \pm G'_i$ and parity violation would indeed be maximal. This situation has been described by saying that neutrinos are like vampires in Dracula's stories: if they were to look to themselves into a mirror they would be unable to see their reflected images.

The actual helicity of the neutrino was measured by Goldhaber et al.⁸ The experiment consisted in observing the K -electron capture in ^{152}Eu ($J = 0$) which produced $^{152}\text{Sm}^*$ ($J = 1$) plus a neutrino. This excited nucleus then decayed into ^{152}Sm ($J = 0$) + γ . Hence the measurement of the polarization of the photon gave the required information on the helicity of the neutrino emitted initially. The conclusion was that '...Our results seem compatible with ... 100% negative helicity for the neutrinos', i.e. that the neutrinos are left handed particles.

This paved the road for the $V - A$ theory of weak interactions advanced by Feynman and Gell Mann, and Marshak and Soudarshan⁹, which stated that weak interactions only involved vector and axial vector currents, in the combination $V - A$ which only allows the coupling to left handed fields, i.e.

$$J_\mu = \bar{e}_L\gamma_\mu\nu_L + \bar{n}_L\gamma_\mu p_L$$

with $H = (G_F/\sqrt{2})J_\mu^\dagger J^\mu$. This interaction also predicted the existence of purely leptonic weak charged currents, e.g. $\nu + e \rightarrow \nu + e$, to be experimen-

tally observed much later^a.

The current involving nucleons is actually not exactly $\propto \gamma_\mu(1-\gamma_5)$ (only the interaction at the quark level has this form), but is instead $\propto \gamma_\mu(g_V - g_A\gamma_5)$. The vector coupling remains however unrenormalised ($g_V = 1$) due to the so called conserved vector current hypothesis (CVC), which states that the vector part of the weak hadronic charged currents ($J_\mu^\pm \propto \bar{\Psi}\gamma_\mu\tau^\pm\Psi$, with τ^\pm the raising and lowering operators in the isospin space $\Psi^T = (p, n)$) together with the isovector part of the electromagnetic current (i.e. the term proportional to τ_3 in the decomposition $J_\mu^{em} \propto \bar{\Psi}\gamma_\mu(1 + \tau_3)\Psi$) form an isospin triplet of conserved currents. On the other hand, the axial vector hadronic current is not protected from strong interaction renormalization effects and hence g_A does not remain equal to unity. The measured value, using for instance the lifetime of the neutron, is $g_A = 1.27$, so that at the nucleonic level the charged current weak interactions are actually “ $V - 1.27A$ ”.

With the present understanding of weak interactions, we know that the clever idea to explain parity violation as due to the non-existence of one of the neutrino chiralities (the right handed one) was completely wrong, although it led to major advances in the theory and ultimately to the correct interaction. Today we understand that the parity violation is a property of the gauge boson (the W) responsible for the gauge interaction, which couples only to the left handed fields, and not due to the absence of right handed fields. For instance, in the quark sector both left and right chiralities exist, but parity is violated because the right handed fields are singlets for the weak charged currents.

1.3. *The trilogy:*

In 1947 the muon was discovered in cosmic rays by Anderson and Neddermeyer. This particle was just a heavier copy of the electron, and as was suggested by Pontecorvo, it also had weak interactions $\mu + p \rightarrow n + \nu$ with the same universal strength G_F . Hincks, Pontecorvo and Steinberger showed that the muon was decaying to three particles, $\mu \rightarrow e\nu\nu$, and the question arose whether the two emitted neutrinos were similar or not. It

^aA curious fact was that the new theory predicted a cross section for the inverse beta decay a factor of two larger than the Bethe and Peierls original result, which had already been confirmed in 1956 to the 5% accuracy by Reines and Cowan. However, in an improved experiment in 1959 Reines and Cowan found a value consistent with the new prediction, what shows that many times when the experiment agrees with the theory accepted at the moment the errors tend to be underestimated.

was then shown by Feinberg¹⁰ that, assuming the two particles were of the same kind, weak interactions couldn't be mediated by gauge bosons (an hypothesis suggested in 1938 by Klein). The reasoning was that if the two neutrinos were equal, it would be possible to join the two neutrino lines and attach a photon to the virtual charged gauge boson (W) or to the external legs^b, so as to generate a diagram for the radiative decay $\mu \rightarrow e\gamma$. The resulting branching ratio would be larger than 10^{-5} and was hence already excluded at that time. This was probably the first use of 'rare decays' to constrain the properties of new particles.

The correct explanation for the absence of the radiative decay was put forward by Lee and Yang, who suggested that the two neutrinos emitted in the muon decay had different flavour, i.e. $\mu \rightarrow e + \nu_e + \nu_\mu$, and hence it was not possible to join the two neutrino lines to draw the radiative decay diagram. This was confirmed at Brookhaven in the first accelerator neutrino experiment¹¹. They used an almost pure $\bar{\nu}_\mu$ beam, something which can be obtained from charged pion decays, since the $V - A$ theory implies that $\Gamma(\pi \rightarrow \ell + \bar{\nu}_\ell) \propto m_\ell^2$, i.e. this process requires a chirality flip in the final lepton line which strongly suppresses the decays $\pi \rightarrow e + \bar{\nu}_e$. Putting a detector in front of this beam they were able to observe the process $\bar{\nu} + p \rightarrow n + \mu^+$, but no production of positrons, what proved that the neutrinos produced in a weak decay in association with a muon were not the same as those produced in a beta decay (in association with an electron). Notice that although the neutrino fluxes are much smaller at accelerators than at reactors, their higher energies make their detection feasible due to the larger cross sections ($\sigma \propto E^2$ for $E \ll m_p$, and $\sigma \propto E$ for $E \gtrsim m_p$).

In 1975 the third charged lepton was discovered by Perl at SLAC, and being just a heavier copy of the electron and the muon, it was concluded that a third neutrino flavour had also to exist. The direct detection of the τ neutrino has been achieved by the DONUT experiment at Fermilab, looking at the short τ tracks produced by the interaction of a ν_τ emitted in the decay of a heavy meson (containing a b quark) produced in a beam dump. Furthermore, we know today that the number of light weakly interacting neutrinos is precisely three (see below), so that the proliferation of neutrino species seems to be now under control.

^bthis reasoning would have actually also excluded a purely leptonic generalisation of a Fermi's theory to describe the muon decay.

1.4. The gauge theory:

As was just mentioned, Klein had suggested that the short range charged current weak interaction could be due to the exchange of a heavy charged vector boson, the W^\pm . This boson exchange would look at small momentum transfers ($Q^2 \ll M_W^2$) as the non renormalisable four fermion interactions discussed before. If the gauge interaction is described by the Lagrangian $\mathcal{L} = -(g/\sqrt{2})J_\mu W^\mu + h.c.$, from the low energy limit one can identify the Fermi coupling as $G_F = \sqrt{2}g^2/(8M_W^2)$. In the sixties, Glashow, Salam and Weinberg showed that it was possible to write down a unified description of electromagnetic and weak interactions with a gauge theory based in the group $SU(2)_L \times U(1)_Y$ (weak isospin \times hypercharge, with the electric charge being $Q = T_3 + Y$), which was spontaneously broken at the weak scale down to the electromagnetic $U(1)_{em}$. This (nowadays standard) model involves the three gauge bosons in the adjoint of $SU(2)$, V_i^μ (with $i = 1, 2, 3$), and the hypercharge gauge field B^μ , so that the starting Lagrangian is

$$\mathcal{L} = -g \sum_{i=1}^3 J_\mu^i V_i^\mu - g' J_\mu^Y B^\mu + h.c.,$$

with $J_\mu^i \equiv \sum_a \bar{\Psi}_{aL} \gamma_\mu (\tau_i/2) \Psi_{aL}$. The left handed leptonic and quark isospin doublets are $\Psi^T = (\nu_{eL}, e_L)$ and (u_L, d_L) for the first generation (and similar ones for the other two heavier generations) and the right handed fields are $SU(2)$ singlets. The hypercharge current is obtained by summing over both left and right handed fermion chiralities and is $J_\mu^Y \equiv \sum_a Y_a \bar{\Psi}_a \gamma_\mu \Psi_a$.

After the electroweak breaking one can identify the weak charged currents with $J^\pm = J^1 \pm iJ^2$, which couple to the W boson $W^\pm = (V^1 \mp iV^2)/\sqrt{2}$, and the two neutral vector bosons V^3 and B will now combine through a rotation by the weak mixing angle θ_W (with $\text{tg}\theta_W = g'/g$), to give

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} c\theta_W & s\theta_W \\ -s\theta_W & c\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ V_\mu^3 \end{pmatrix}. \quad (1)$$

We see that the broken theory has now, besides the massless photon field A_μ , an additional neutral vector boson, the heavy Z_μ , whose mass turns out to be related to the W boson mass through $s^2\theta_W = 1 - (M_W^2/M_Z^2)$. The electromagnetic and neutral weak currents are given by

$$J_\mu^{em} = J_\mu^Y + J_\mu^3$$

$$J_\mu^0 = J_\mu^3 - s^2\theta_W J_\mu^{em},$$

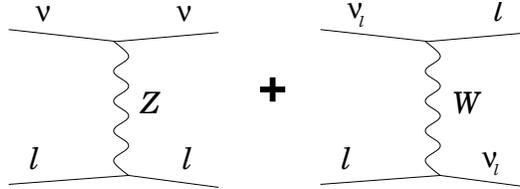


Figure 1. Neutral and charged current contributions to neutrino lepton scattering.

with the electromagnetic coupling being $e = g s\theta_W$.

The great success of this model came in 1973 with the experimental observation of the weak neutral currents using muon neutrino beams at CERN (Gargamelle) and Fermilab, using the elastic process $\nu_\mu e \rightarrow \nu_\mu e$. The semileptonic processes $\nu N \rightarrow \nu X$ were also studied and the comparison of neutral and charged current rates provided a measure of the weak mixing angle. From the theoretical side t'Hooft proved the renormalisability of the theory, so that the computation of radiative corrections became also meaningful.

The Hamiltonian for the leptonic weak interactions $\nu_\ell + \ell' \rightarrow \nu_\ell + \ell'$ can be obtained, using the Standard Model just presented, from the two diagrams in figure 1. In the low energy limit ($Q^2 \ll M_W^2, M_Z^2$), it is just given by

$$H_{\nu_\ell \ell'} = \sqrt{2}G_F [\bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \nu_\ell] [\bar{\ell}' \gamma^\mu (c_L P_L + c_R P_R) \ell']$$

where the left and right couplings are $c_L = \delta_{\ell\ell'} + s^2\theta_W - 0.5$ and $c_R = s^2\theta_W$. The $\delta_{\ell\ell'}$ term in c_L is due to the charged current diagram, which clearly only appears when $\ell = \ell'$. On the other hand, one sees that due to the B component in the Z boson, the weak neutral currents also couple to the charged lepton right handed chiralities (i.e. $c_R \neq 0$). This interaction leads to the cross section (for $E_\nu \gg m_{\ell'}$)

$$\sigma(\nu + \ell \rightarrow \nu + \ell) = \frac{2G_F^2}{\pi} m_\ell E_\nu \left[c_L^2 + \frac{c_R^2}{3} \right],$$

and a similar expression with $c_L \leftrightarrow c_R$ for antineutrinos. Hence, we have the following relations for the neutrino elastic scatterings off electrons

$$\sigma(\nu_e e) \simeq 9 \times 10^{-44} \text{cm}^2 \left(\frac{E_\nu}{10 \text{ MeV}} \right) \simeq 2.5\sigma(\bar{\nu}_e e) \simeq 6\sigma(\nu_{\mu,\tau} e) \simeq 7\sigma(\bar{\nu}_{\mu,\tau} e).$$

Regarding the angular distribution of the electron momentum with respect to the incident neutrino direction, in the center of mass system of the

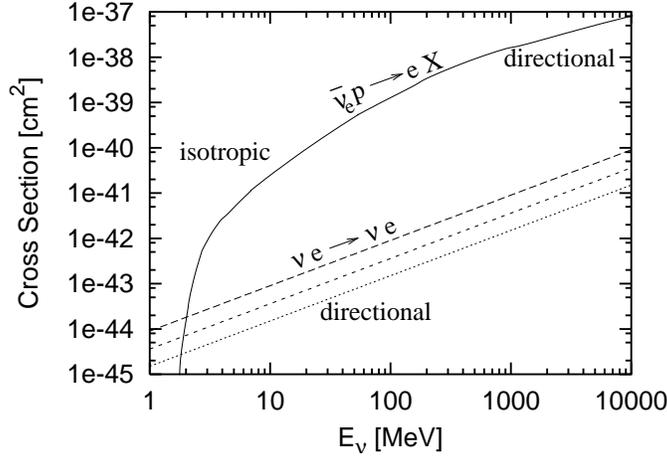


Figure 2. Neutrino nucleon and neutrino lepton cross sections (the three lines correspond, from top to bottom, to the ν_e , $\bar{\nu}_e$ and $\nu_{\mu,\tau}$ cross sections with electrons).

process $d\sigma(\nu_e e)/d\cos\theta \propto 1 + 0.1[(1 + \cos\theta)/2]^2$, and it is hence almost isotropic. However, due to the boost to the laboratory system, there will be a significant correlation between the neutrino and electron momenta for $E_\nu \gg \text{MeV}$, and this actually allows to do astronomy with neutrinos. For instance, water cherenkov detectors such as Superkamiokande detect solar neutrinos using this process, and have been able to reconstruct a picture of the Sun with neutrinos. It will turn also to be relevant for the study of neutrino oscillations that these kind of detectors are six times more sensitive to electron type neutrinos than to the other two neutrino flavours.

Considering now the neutrino nucleon interactions, one has at low energies ($1 \text{ MeV} < E_\nu < 50 \text{ MeV}$) the cross section for the quasi-elastic process^c

$$\sigma(\nu_e n \rightarrow pe) \simeq \sigma(\bar{\nu}_e p \rightarrow ne^+) \simeq \frac{G_F^2}{\pi} c^2 \theta_C (g_V^2 + 3g_A^2) E_\nu^2,$$

where we have now introduced the Cabibbo mixing angle θ_C which relates, if we ignore the third family, the quark flavour eigenstates q^0 to the mass eigenstates q , i.e. $d^0 = c\theta_C d + s\theta_C s$ and $s^0 = -s\theta_C d + c\theta_C s$ (choosing a flavour basis so that the up type quark flavour and mass eigenstates coincide).

^cActually for the $\bar{\nu}_e p$ CC interaction, the threshold energy is $E_\nu^{th} \simeq m_n - m_p + m_e \simeq 1.8 \text{ MeV}$.

At $E_\nu \gtrsim 50$ MeV, the nucleon no longer looks like a point-like object for the neutrinos, and hence the vector (v_μ) and axial (a_μ) hadronic currents involve now momentum dependent form factors, i.e.

$$\langle N(p') | v_\mu | N(p) \rangle = \bar{u}(p') [\gamma_\mu F_V + \frac{i}{2m_N} \sigma_{\mu\nu} q^\nu F_W] u(p)$$

$$\langle N(p') | a_\mu | N(p) \rangle = \bar{u}(p') [\gamma_\mu \gamma_5 F_A + \frac{\gamma_5}{m_N} q_\mu F_P] u(p),$$

where $F_V(q^2)$ can be measured using electromagnetic processes and the CVC relation $F_V = F_V^{em,p} - F_V^{em,n}$ (i.e. as the difference between the proton and neutron electromagnetic vector form factors). Clearly $F_V(0) = 1$ and $F_A(0) = 1.27$, while F_W is related to the anomalous magnetic moments of the nucleons. The q^2 dependence has the effect of significantly flattening the cross section. In the deep inelastic regime, $E_\nu \gtrsim$ GeV, the neutrinos interact directly with the quark constituents. The cross section in this regime grows linearly with energy, and this provided an important test of the parton model. The main characteristics of the neutrino cross section just discussed are depicted in figure 2. For even larger energies, the gauge boson propagators enter into the play (e.g. $1/M_W^2 \rightarrow 1/q^2$) and the growth of the cross section is less pronounced above 10 TeV ($\sigma \propto E^{0.36}$).

The most important test of the standard model came with the direct production of the W^\pm and Z gauge bosons at CERN in 1984, and with the precision measurements achieved with the Z factories LEP and SLC after 1989. These e^+e^- colliders working at and around the Z resonance ($s = M_Z^2 = (91 \text{ GeV})^2$) turned out to be also crucial for neutrino physics, since studying the shape of the $e^+e^- \rightarrow f\bar{f}$ cross section near the resonance, which has the Breit-Wigner form

$$\sigma \simeq \frac{12\pi\Gamma_e\Gamma_f}{M_Z^2} \frac{s}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2},$$

it becomes possible to determine the total Z width Γ_Z . This width is just the sum of all possible partial widths, i.e.

$$\Gamma_Z = \sum_f \Gamma_{Z \rightarrow f\bar{f}} = \Gamma_{vis} + \Gamma_{inv}.$$

The visible (i.e. involving charged leptons and quarks) width Γ_{vis} can be measured directly, and hence one can infer a value for the invisible width Γ_{inv} . Since in the standard model this last arises from the decays $Z \rightarrow \nu_i \bar{\nu}_i$, whose expected rate for decays into a given neutrino flavour is $\Gamma_{Z \rightarrow \nu\bar{\nu}}^{th} = 167 \text{ MeV}$, one can finally obtain the number of neutrinos coupling

to the Z as $N_\nu = \Gamma_{inv}/\Gamma_{Z \rightarrow \nu\bar{\nu}}^{th}$. The present best value for this quantity is $N_\nu = 2.994 \pm 0.012$, giving then a strong support to the three generation standard model.

Going through the history of the neutrinos we have seen that they have been extremely useful to understand the standard model. On the contrary, the standard model is of little help to understand the neutrinos. Since in the standard model there is no need for ν_R , neutrinos are massless in this theory. There is however no deep principle behind this (unlike the masslessness of the photon which is protected by the electromagnetic gauge symmetry), and indeed in many extensions of the standard model neutrinos turn out to be massive. This makes the search for non-zero neutrino masses a very important issue, since it provides a window to look for physics beyond the standard model. Indeed, solid evidence has accumulated in the last years indicating that neutrinos are massive, and this makes the field of neutrino physics even more exciting now than in the long historic period that we have just reviewed.

2. Neutrino masses:

2.1. Dirac or Majorana?

In the standard model, charged leptons (and also quarks) get their masses through their Yukawa couplings to the Higgs doublet field $\phi^T = (\phi_+, \phi_0)$

$$-\mathcal{L}_Y = \lambda \bar{L} \phi \ell_R + h.c. ,$$

where $L^T = (\nu, \ell)_L$ is a lepton doublet and ℓ_R an SU(2) singlet field. When the electroweak symmetry gets broken by the vacuum expectation value of the neutral component of the Higgs field $\langle \phi_0 \rangle = v/\sqrt{2}$ (with $v = 246$ GeV), the following ‘Dirac’ mass term results

$$-\mathcal{L}_m = m_\ell (\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L) = m_\ell \bar{\ell} \ell ,$$

where $m_\ell = \lambda v/\sqrt{2}$ and $\ell = \ell_L + \ell_R$ is the Dirac spinor field. This mass term is clearly invariant under the $U(1)$ transformation $\ell \rightarrow \exp(i\alpha)\ell$, which corresponds to the lepton number (and actually in this case also to the electromagnetic gauge invariance). From the observed fermion masses, one concludes that the Yukawa couplings range from $\lambda_t \simeq 1$ for the top quark up to $\lambda_e \simeq 10^{-5}$ for the electron.

Notice that the mass terms always couple fields with opposite chiralities, i.e. requires a $L \leftrightarrow R$ transition. Since in the standard model the right

handed neutrinos are not introduced, it is not possible to write a Dirac mass term, and hence the neutrino results massless. Clearly the simplest way to give the neutrino a mass would be to introduce the right handed fields just for this purpose (having no gauge interactions, these sterile states would be essentially undetectable and unproduceable). Although this is a logical possibility, it has the ugly feature that in order to get reasonable neutrino masses, below the eV, would require unnaturally small Yukawa couplings ($\lambda_\nu < 10^{-11}$). Fortunately it turns out that neutrinos are also very special particles in that, being neutral, there are other ways to provide them a mass. Furthermore, in some scenarios it becomes also possible to get a natural understanding of why neutrino masses are so much smaller than the charged fermion masses.

The new idea is that the left handed neutrino field actually involves two degrees of freedom, the left handed neutrino associated with the positive beta decay (i.e. emitted in association with a positron) and the other one being the right handed ‘anti’-neutrino emitted in the negative beta decays (i.e. emitted in association with an electron). It may then be possible to write down a mass term using just these two degrees of freedom and involving the required $L \leftrightarrow R$ transition. This possibility was first suggested by Majorana in 1937, in a paper named ‘Symmetric theory of the electron and positron’, and devoted mainly to the problem of getting rid of the negative energy sea of the Dirac equation¹². As a side product, he found that for neutral particles there was ‘no more any reason to presume the existence of antiparticles’, and that ‘it was possible to modify the theory of beta emission, both positive and negative, so that it came always associated with the emission of a neutrino’. The spinor field associated to this formalism was then named in his honor a Majorana spinor.

To see how this works it is necessary to introduce the so called antiparticle field, $\psi^c \equiv C\bar{\psi}^T = C\gamma_0^T\psi^*$. The charge conjugation matrix C has to satisfy $C\gamma_\mu C^{-1} = -\gamma_\mu^T$, so that for instance the Dirac equation for a charged fermion in the presence of an electromagnetic field, $(i\cancel{\partial} - e\cancel{A} - m)\psi = 0$ implies that $(i\cancel{\partial} + e\cancel{A} - m)\psi^c = 0$, i.e. that the antiparticle field has the same mass but charges opposite to those of the particle field. Since for a chiral projection one can show that $(\psi_L)^c = (P_L\psi)^c = P_R\psi^c = (\psi^c)_R$, i.e. this conjugation changes the chirality of the field, one has that ψ^c is related to the CP conjugate of ψ . Notice that $(\psi_L)^c$ describes exactly the same two degrees of freedom described by ψ_L , but somehow using a CP reflected formalism. For instance for the neutrinos, the ν_L operator annihilates the left handed neutrino and creates the right handed antineutrino, while the

$(\nu_L)^c$ operator annihilates the right handed antineutrino and creates the left handed neutrino.

We can then now write the advertised Majorana mass term, as

$$-\mathcal{L}_M = \frac{1}{2}m[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L].$$

This mass term has the required Lorentz structure (i.e. the $L \leftrightarrow R$ transition) but one can see that it does not preserve any $U(1)$ phase symmetry, i.e. it violates the so called lepton number by two units. If we introduce the Majorana field $\nu \equiv \nu_L + (\nu_L)^c$, which under conjugation transforms into itself ($\nu^c = \nu$), the mass term becomes just $\mathcal{L}_M = -m\bar{\nu}\nu/2$.

Up to now we have introduced the Majorana mass by hand, contrary to the case of the charged fermions where it arose from a Yukawa coupling in a spontaneously broken theory. To follow the same procedure with the neutrinos presents however a difficulty, because the standard model neutrinos belong to $SU(2)$ doublets, and hence to write an electroweak singlet Yukawa coupling it is necessary to introduce an $SU(2)$ triplet Higgs field $\vec{\Delta}$ (something which is not particularly attractive). The coupling $\mathcal{L} \propto \overline{L^c}\vec{\sigma}L\cdot\vec{\Delta}$ would then lead to the Majorana mass term after the neutral component of the scalar gets a VEV. Alternatively, the Majorana mass term could be a loop effect in models where the neutrinos have lepton number violating couplings to new scalars, as in the so-called Zee models or in the supersymmetric models with R parity violation. These models have as interesting features that the masses are naturally suppressed by the loop, and they are attractive also if one looks for scenarios where the neutrinos have relatively large dipole moments, since a photon can be attached to the charged particles in the loop.

However, by far the nicest possibility to give neutrinos a mass is the so-called see-saw model¹³. In this scenario, which naturally occurs in grand unified models such as $SO(10)$, one introduces the $SU(2)$ singlet right handed neutrinos. One has now not only the ordinary Dirac mass term, but also a Majorana mass for the singlets which is generated by the VEV of an $SU(2)$ singlet Higgs, whose natural scale is the scale of breaking of the grand unified group, i.e. in the range 10^{12} – 10^{16} GeV. Hence the Lagrangian will contain

$$\mathcal{L}_M = \frac{1}{2}\overline{(\nu_L, (N_R)^c)} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ N_R \end{pmatrix} + h.c..$$

The mass eigenstates are two Majorana fields with masses $m_{light} \simeq m_D^2/M$ and $m_{heavy} \simeq M$. Since $m_D/M \ll 1$, we see that $m_{light} \ll m_D$, and hence

the lightness of the known neutrinos is here related to the heaviness of the sterile states N_R .

If we actually introduce one singlet neutrino per family, the mass terms in eq. (2.1) are 3×3 matrices. Notice that if m_D is similar to the up-type quark masses, as happens in $SO(10)$, one would have $m_{\nu_3} \sim m_t^2/M \simeq 0.04 \text{ eV}(10^{15} \text{ GeV}/M)$. It is clear then that in these scenarios the observation of neutrino masses near 0.1 eV would point out to new physics at about the GUT scale, while for $m_D \sim \text{GeV}$ this would correspond to singlet neutrino masses at an intermediate scale $M \sim 10^{10}\text{--}10^{12} \text{ GeV}$, also of great theoretical interest.

2.2. Neutrino mixing and oscillations:

If neutrinos are massive, there is no reason to expect that the mass eigenstates (ν_k , with $k = 1, 2, 3$) would coincide with the flavour (gauge) eigenstates (ν_α , with $\alpha = e, \mu, \tau$, where we are adopting the flavor basis such that for the charged leptons the flavor eigenstates coincide with the mass eigenstates), and hence, in the same way that quark states are mixed through the Cabibbo, Kobayashi and Maskawa matrix, neutrinos would be related through the Maki, Nakagawa and Sakita ¹⁴ (and Pontecorvo) mixing matrix, i.e. $\nu_\alpha = V_{\alpha k}^* \nu_k$. This matrix can be parametrized as ($c_{12} \equiv \cos \theta_{12}$, etc.)

$$V = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The phases $\alpha_{1,2}$ here cannot be removed by a rephasing of the fields (as is done for quarks) if the neutrinos are Majorana particles, since such rotations would then introduce a complex phase in the neutrino masses.

The possibility that neutrino flavour eigenstates be a superposition of mass eigenstates allows for the phenomenon of neutrino oscillations. This is a quantum mechanical interference effect (and as such it is sensitive to quite small masses) and arises because different mass eigenstates propagate differently, and hence the flavor composition of a state can change with time.

To see this consider a flavour eigenstate neutrino ν_α with momentum p produced at time $t = 0$ (e.g. a ν_μ produced in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$). The initial state is then

$$|\nu_\alpha\rangle = \sum_k V_{\alpha k}^* |\nu_k\rangle.$$

We know that the mass eigenstates evolve with time according to $|\nu_k(t, x)\rangle = \exp[i(px - E_k t)]|\nu_k\rangle$. In the relativistic limit relevant for neutrinos, one has that $E_k = \sqrt{p^2 + m_k^2} \simeq p + m_k^2/2E$, and thus the different mass eigenstates will acquire different phases as they propagate. Hence, the probability of observing a flavour ν_β at time t is just

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu(t) \rangle|^2 = \left| \sum_k V_{\alpha k}^* e^{-i \frac{m_k^2}{2E} t} V_{\beta k} \right|^2.$$

Taking into account the explicit expression for V , it is easy to convince oneself that the Majorana phases $\alpha_{1,2}$ do not enter into the oscillation probability, and hence oscillation phenomena cannot tell whether neutrinos are Dirac or Majorana particles.

In the case of two generations, V can be taken just as a rotation R_θ with mixing angle θ , so that one has

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 x}{4E} \right),$$

which depends on the squared mass difference $\Delta m^2 = m_2^2 - m_1^2$, since this is what gives the phase difference in the propagation of the mass eigenstates. Hence, the amplitude of the flavour oscillations is given by $\sin^2 2\theta$ and the oscillation length of the modulation is $L_{osc} \equiv 4\pi E / \Delta m^2 \simeq 2.5 \text{ m} E[\text{MeV}] / \Delta m^2[\text{eV}^2]$. We see then that neutrinos will typically oscillate with a macroscopic wavelength. For instance, putting a detector at $\sim 1 \text{ km}$ from a reactor (such as in the CHOOZ experiment) allows to test oscillations of ν_e 's to another flavour (or into a singlet neutrino) down to $\Delta m^2 \sim 10^{-3} \text{ eV}^2$ if $\sin^2 2\theta$ is not too small (≥ 0.1). These kind of experiments look essentially for the disappearance of the reactor ν_e 's, i.e. to a reduction in the original ν_e flux. When one uses neutrino beams from accelerators, it becomes possible also to study the disappearance of muon neutrinos into another flavor, and also the appearance of a flavour different from the original one, with the advantage that one becomes sensitive to very small oscillation amplitudes (i.e. small $\sin^2 2\theta$ values), since the observation of only a few events is enough to establish a positive signal. At present there is one experiment (LSND) claiming a positive signal of $\nu_\mu \rightarrow \nu_e$ conversion, but this highly suspected result is expected to be clarified unambiguously by the MINIBOONE experiment at Fermilab during 2005. The appearance of ν_τ 's out of a ν_μ beam was searched by CHORUS and NOMAD at CERN without success, but these experiments were only sensitive to squared mass differences larger than $\sim \text{eV}^2$. There are two experiments which have obtained

recently solid evidence of neutrino oscillations (K2K and Kamland), but let's however, following the historical evolution, start with the discussion of solar and atmospheric neutrinos and the clues they have given in favor of non-vanishing neutrino masses.

2.3. *Solar neutrinos and oscillations in matter:*

The Sun gets its energy from the fusion reactions taking place in its interior, where essentially four protons combine to form a He nucleus. By charge conservation this has to be accompanied by the emission of two positrons and, by lepton number conservation in the weak processes, two ν_e 's have to be produced. This fusion liberates 27 MeV of energy, which is eventually emitted mainly (97%) as photons and the rest (3%) as neutrinos. Knowing the energy flux of the solar radiation reaching us ($k_{\odot} \simeq 1.5 \text{ kW/m}^2$), it is then simple to estimate that the solar neutrino flux at Earth is $\Phi_{\nu} \simeq 2k_{\odot}/27 \text{ MeV} \simeq 6 \times 10^{10} \nu_e/\text{cm}^2\text{s}$, which is a very large number indeed. Since there are many possible paths for the four protons to lead to an He nucleus, the solar neutrino spectrum consists of different components: the so-called *pp* neutrinos are the more abundant, but have very small energies ($< 0.4 \text{ MeV}$), the ${}^8\text{B}$ neutrinos are the more energetic ones ($< 14 \text{ MeV}$) but are much less in number, there are also some monochromatic lines (${}^7\text{Be}$ and *pep* neutrinos), and then the *CNO* and *hep* neutrinos.

Many experiments have looked for these solar neutrinos: the radiochemical experiments with ${}^{37}\text{Cl}$ at Homestake and with gallium at SAGE, GALLEX and GNO, and the water Cherenkov real time detectors (Super-) Kamiokande and more recently the heavy water Sudbury Neutrino Observatory (SNO)^d. The result which has puzzled physicists for almost thirty years is that only between 1/2 to 1/3 of the expected fluxes were observed. Remarkably, Pontecorvo¹⁵ noticed even before the first observation of solar neutrinos by Davies that neutrino oscillations could reduce the expected rates. We note that the oscillation length of solar neutrinos ($E \sim 0.1\text{--}10 \text{ MeV}$) is of the order of 1 AU for $\Delta m^2 \sim 10^{-11} \text{ eV}^2$, and hence even those tiny neutrino masses could have had observable effects if the mixing angles were large (this would be the 'just so' solution to the solar neutrino problem). Much more interesting became the possibility of explaining the puzzle by resonantly enhanced oscillations of neutrinos as they propagate outwards through the Sun. Indeed, the solar medium affects ν_e 's differ-

^dSee the Neutrino 2004 homepage at <http://neutrino2004.in2p3.fr> for this year's results.

ently than $\nu_{\mu,\tau}$'s (since only the first interact through charged currents with the electrons present), and this modifies the oscillations in a beautiful way through an interplay of neutrino mixings and matter effects, in the so called MSW effect¹⁶.

To see how this effect works it is convenient to write the effective CC interaction (after a Fierz rearrangement) as

$$H^{CC} = \sqrt{2}G_F \bar{e}\gamma_\mu(1 - \gamma_5)e \bar{\nu}_{eL}\gamma^\mu\nu_{eL}. \quad (2)$$

Since the electrons in a normal medium (such as the Sun or the interior of the Earth) are non-relativistic, one can see that $\bar{e}\gamma_\mu e \rightarrow (N_e, \vec{0})$, where N_e is the electron density, while for the axial vector part one gets $\bar{e}\gamma_\mu\gamma_5 e \rightarrow (0, \vec{S}_e)$, which vanishes for an unpolarised medium, as is the case of interest here. This means that the electron neutrinos will feel a potential

$$V_{CC} = \langle e\nu_e | H^{CC} | e\nu_e \rangle \simeq \sqrt{2}G_F N_e \quad (3)$$

(and for the antineutrinos the potential will have a minus sign in front). The evolution of the neutrino states will hence be determined by a Schroedinger like equation of the form (for the case of just two flavor mixing, i.e. $\alpha = \mu$ or τ)

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} = \left\{ R_\theta \begin{bmatrix} p + \frac{m_1^2}{2E} & 0 \\ 0 & p + \frac{m_2^2}{2E} \end{bmatrix} R_\theta^T + \begin{bmatrix} \sqrt{2}G_F N_e & 0 \\ 0 & 0 \end{bmatrix} + NC \right\} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} \quad (4)$$

where θ is the vacuum mixing angle and

$$R_\theta \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (5)$$

The terms indicated as NC correspond to the effective potential induced by the neutral current interactions of the neutrinos with the medium, but since these are flavor blind, this term is proportional to the identity matrix and hence does not affect the flavor oscillations, so that we can ignore it in the following (these terms could be relevant e.g. when studying oscillations into sterile neutrinos, which unlike the active ones do not feel NC interactions).

To solve this equation it is convenient to introduce a mixing angle in matter θ_m and define the neutrino matter eigenstates

$$\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} \equiv R_{\theta_m}^T \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} \quad (6)$$

such that the evolution equation becomes

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} = \left\{ \frac{1}{4E} R_{\theta_m} \begin{bmatrix} -\Delta\mu^2 & 0 \\ 0 & \Delta\mu^2 \end{bmatrix} R_{\theta_m}^T + \lambda I \right\} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} \quad (7)$$

where the diagonal term λI is again irrelevant for oscillations. It is simple to show that to have such ‘diagonalisation’ of the effective Hamiltonian one needs

$$\Delta\mu^2 = \Delta m^2 \sqrt{(a - c2\theta)^2 + s^2 2\theta} \quad (8)$$

$$s^2 \theta_m = \frac{s^2 2\theta}{(c2\theta - a)^2 + s^2 2\theta} \quad (9)$$

where $a \equiv 2\sqrt{2}G_F N_e E_\nu / \Delta m^2$ is just the ratio between the effective CC matter potential and the energy splitting between the two vacuum mass eigenstates. It is clear then that there will be a resonant behaviour for $a = c2\theta$, i.e. for

$$\Delta m^2 c2\theta = 2\sqrt{2}G_F N_e E_\nu \simeq \left(\frac{Y_e}{0.5}\right) \left(\frac{E_\nu}{10 \text{ MeV}}\right) \frac{\rho}{100 \text{ g/cm}^3} 10^{-4} \text{ eV}^2, \quad (10)$$

where for the second relation we used that $N_e = Y_e \rho / m_p$, with $Y_e \equiv N_e / (N_n + N_p)$ (for the Sun $Y_e \sim 0.7-0.8$). One can see that at the resonance the matter mixing angle becomes maximal, i.e. $\theta_m = \pi/4$, while at densities much larger than the resonant one it becomes $\sim \pi/2$ (i.e. one gets for densities much larger than the resonance one that $\nu_e \simeq \nu_2^m$).

In the case of the Sun, one has that the density decreases approximately exponentially

$$\rho \simeq 10^2 \frac{\text{g}}{\text{cm}^3} \exp(-r/h) \quad (11)$$

with the scale height being $h \simeq 0.1R_\odot$ in terms of the solar radius R_\odot . Hence, solar neutrinos, which are produced near the center, will cross a resonance in their way out only if $\Delta m^2 < 10^{-4} \text{ eV}^2$, and moreover only if $\Delta m^2 > 0$. For the opposite sign of Δm^2 only antineutrinos, which are however not produced in fusion processes, could meet a resonance in normal matter^e.

One can also associate a width δ_R to the resonance, corresponding to the density for which $|a - c2\theta| \simeq s2\theta$, i.e. $|da/dr|_R \delta_R \simeq s2\theta$. This leads to $\delta_R \simeq h \text{ tg}2\theta$. This width is useful to characterize the two basic regimes of resonant flavor conversions, which are the adiabatic one, taking place when

^eAlternatively, one can stick to positive values of Δm^2 and consider vacuum mixing angles in the range $0 \leq \theta \leq \pi/2$, with the range $\pi/4 \leq \theta \leq \pi/2$ sometimes called the ‘dark side’ of the parameter space.

the oscillation length in matter is much smaller than the resonance width, i.e. for

$$\frac{4\pi E_\nu}{\Delta\mu^2|_R} = \frac{4\pi E_\nu}{\Delta m^2 s 2\theta} < h \operatorname{tg} 2\theta, \quad (12)$$

and the opposite one, which is called non-adiabatic, for which the resonance is so narrow that the oscillating neutrinos effectively don't see it and hence no special flavor transition occurs at the resonant crossing. The adiabatic condition can be rewritten as

$$\frac{s^2 2\theta}{c 2\theta} > \left(\frac{E_\nu}{10 \text{ MeV}} \right) \left(\frac{6 \times 10^{-8} \text{ eV}^2}{\Delta m^2} \right). \quad (13)$$

To better understand the flavor transition during resonance crossing, it proves convenient to write down the evolution equation for the matter mass eigenstates, which is easily obtained as (ignoring terms proportional to the identity)

$$i \frac{d}{dx} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} = \begin{pmatrix} -\frac{\Delta\mu^2}{4E} & -i \frac{d\theta_m}{dx} \\ i \frac{d\theta_m}{dx} & \frac{\Delta\mu^2}{4E} \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} \quad (14)$$

We see that in the adiabatic case, the off diagonal terms in this equation are negligible and hence during the resonance crossing the matter mass eigenstates remain themselves so that the flavor of the neutrinos changes just following the change on the matter mixing angle with the varying electron density. This adiabatic behaviour is also relevant for the propagation of neutrinos in a medium of constant density (as is sometimes a good approximation for the propagation through the Earth), and in this case the matter effects just change the mixing angle and the frequency of the oscillations among neutrinos. When the propagation is non-adiabatic, the off-diagonal terms in Eq. (14) induce transitions between the different matter mass eigenstates as the resonance is crossed. Indeed the probability of jumping from one eigenstate to the other during resonance crossing for an exponential density profile can be written as

$$P_c(\nu_1^m \rightarrow \nu_2^m) = \frac{\exp(-\gamma \sin^2 \theta) - \exp(-\gamma)}{1 - \exp(-\gamma)}, \quad (15)$$

where the adiabaticity parameter is $\gamma \equiv \pi h \Delta m^2 / E$ (notice that for an electron density varying in a more general way, not just exponentially, it is usually a good approximation to replace h in the above formulas by $|(dN_e/dr)/N_e|_R^{-1}$).

The many observations of the solar neutrino fluxes by the different experiments, which having different energy thresholds are sensitive to the oscillation probabilities in different energy ranges (moreover water Cherenkov detectors can measure the neutrino spectrum directly above their thresholds), and also the non-observation of a possible diurnal modulation induced by the matter effects when neutrinos have to cross the Earth before reaching the detectors during night-time observations, have converged over the years towards the so-called large mixing angle (LMA) solution as the one required to account for the solar neutrino observations. This one corresponds to mixing of ν_e with some combination of ν_μ and ν_τ flavors involving a mass splitting between the mass eigenstates of^f $\Delta m_{sol}^2 = +(7.9_{-0.5}^{+0.6}) \times 10^{-5} \text{ eV}^2$ with a mixing angle given by $\tan^2 \theta_{sol} = 0.40_{-0.07}^{+0.10}$. This values imply that the resonance layer is actually at large densities near the center of the Sun, and that it is quite wide, so that matter oscillations are well in the adiabatic regime.

Another crucial result obtained in 2002 was the independent measurement of the CC and NC interactions of the solar neutrinos with the heavy water experiment SNO¹⁷. The result is that the NC rates, which are sensitive to the three flavors of active neutrinos, are indeed consistent with the solar model expectations for ν_e alone in the absence of oscillations, while the CC rates, which are sensitive to the electron neutrinos alone, show the deficit by a factor ~ 3 , indicating that the oscillations have occurred and that they convert electron neutrinos into other active neutrino flavors ($\nu_{\mu,\tau}$).

The last remarkable result that has confirmed this picture has been the observation of oscillations of reactor neutrinos (from a large number of japanese reactors) using a huge 1 kton scintillator detector (KAMLAND), measuring oscillations over distances of $\sim 10^2 \text{ km}$, and the reduction found from expectations just agrees with those resulting from the LMA parameters, and have actually restricted the mass splitting involved to the narrow range just mentioned, as is shown in figure 3 (from the Kamland experiment).

2.4. Atmospheric neutrinos:

When a cosmic ray (proton or nucleus) hits the atmosphere and knocks a nucleus a few tens of km above ground, an hadronic (and electromagnetic)

^fthis number includes also the results from the Kamland experiment¹⁸.

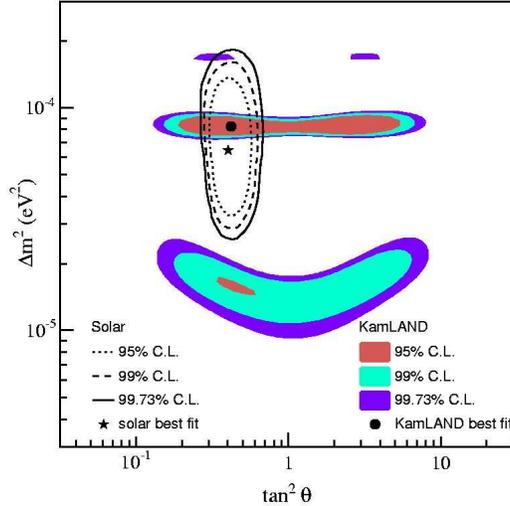


Figure 3. Bounds from the KAMLAND experiment. The region favored by solar neutrino observations is that with unfilled contours.

shower is initiated, in which pions in particular are copiously produced. The charged pion decays are the main source of atmospheric neutrinos through the chain $\pi \rightarrow \mu\nu_\mu \rightarrow e\nu_e\nu_\mu\nu_\mu$. One expects then twice as many ν_μ 's than ν_e 's (actually at very high energies, $E_\nu \gg \text{GeV}$, the parent muons may reach the ground and hence be stopped before decaying, so that the expected ratio $R \equiv (\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ should be even larger than two at high energies). However, the observation of the atmospheric neutrinos by e.g. IMB, Kamioka, Soudan, MACRO and Super-Kamiokande indicates that there is a deficit of muon neutrinos, with $R_{obs}/R_{th} \simeq 2/3$.

More remarkably, the Super-Kamiokande experiment¹⁹ observes a zenith angle dependence indicating that neutrinos coming from above (with pathlengths $d \sim 20$ km) had not enough time to oscillate, especially in the multi-GeV sample for which the neutrino oscillation length is larger, while those from below ($d \sim 13000$ km) have already oscillated (see figure 4). The most plausible explanation for these effects is an oscillation $\nu_\mu \rightarrow \nu_\tau$ with maximal mixing $\sin^2 2\theta_{atm} = 1.00 \pm 0.04$ and $\Delta m^2 \simeq (2.5 \pm 0.4) \times 10^{-3} \text{ eV}^2$, and as shown in fig. 4 (from the Super-Kamiokande experiment) the fit to the observed angular dependence is in excellent agreement with the oscillation hypothesis. Since the electron flux shape is in good agreement with the

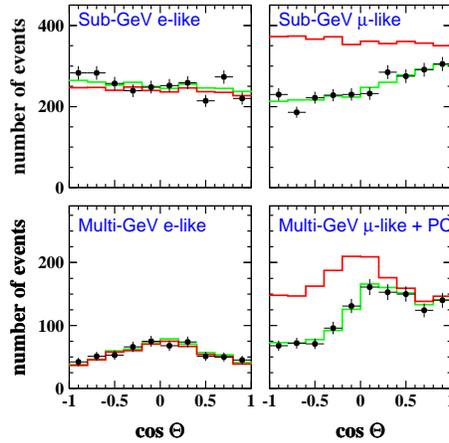


Figure 4. Distribution of the contained and partially contained event data versus cosine of the zenith angle ($\cos \theta = -1$ being up-going, while $+1$ being down-going) for two energy ranges, from Super-Kamiokande data. The solid line corresponds to the expectations with no oscillations, while the lighter line is for $\nu_\mu \rightarrow \nu_\tau$ oscillations with maximal mixing and $\Delta m^2 = 0.003 \text{ eV}^2$.

theoretical predictions[§], this means that the oscillations from $\nu_\mu \rightarrow \nu_e$ can not provide a satisfactory explanation for the anomaly (and furthermore they are also excluded from the negative results of the CHOOZ reactor search for oscillations). On the other hand, oscillations to sterile states would be affected by matter effects (ν_μ and ν_τ are equally affected by neutral current interactions when crossing the Earth, while sterile states are not), and this would modify the angular dependence of the oscillations in a way which is not favored by observations. The oscillations into active states (ν_τ) is also favored by observables which depend on the neutral current interactions, such as the π_0 production in the detector or the ‘multi ring’ events.

An important experiment which has confirmed the oscillation solution to the atmospheric neutrino anomaly is K2K²⁰, consisting of a beam of muon neutrinos sent from KEK to the Super-Kamiokande detector (baseline of

[§]The theoretical uncertainties in the absolute flux normalisation may amount to $\sim 25\%$, but the predictions for the ratio of muon to electron neutrino flavours and for their angular dependence are much more robust.

250 km). The results indicate that there is a deficit of muon neutrinos at the detector (150.9 ± 10 events expected with only 108 observed), consistent with the expectations from the oscillation solution.

It is remarkable that the mixing angle involved seems to be maximal, and this, together with the large mixing angle required in the solar sector, is giving fundamental information about the new physics underlying the origin of the neutrino masses, which seem to be quite different from what is observed in the quark sector.

2.5. *The direct searches for the neutrino mass:*

Already in his original paper on the theory of weak interactions Fermi had noticed that the observed shape of the electron spectrum was suggesting a small mass for the neutrino. The sensitivity to m_{ν_e} in the decay $X \rightarrow X' + e + \bar{\nu}_e$ arises clearly because the larger m_ν , the less available kinetic energy remains for the decay products, and hence the maximum electron energy is reduced. To see this consider the phase space factor of the decay, $d\Gamma \propto d^3p_e d^3p_\nu \propto p_e E_e dE_e p_\nu E_\nu dE_\nu \delta(E_e + E_\nu - E_0)$, with E_0 being the total energy available for the leptons in the decay: $E_0 \simeq M_X - M_{X'}$ (neglecting the nuclear recoil). This leads to a differential electron spectrum proportional to $d\Gamma/dE_e \propto p_e E_e (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_\nu^2}$, whose shape near the endpoint ($E_e \simeq E_0 - m_\nu$) depends on m_ν (actually the slope becomes infinite at the endpoint for $m_\nu \neq 0$, while it vanishes for $m_\nu = 0$).

Since the fraction of events in an interval ΔE_e around the endpoint is $\sim (\Delta E_e/Q)^3$ (where $Q \equiv E_0 - m_e$), to enhance the sensitivity to the neutrino mass it is better to use processes with small Q -values, what makes the tritium the most sensitive nucleus ($Q = 18.6$ keV). Experiments at Mainz and Troitsk have allowed to set the bound $m_{\nu_e} \leq 2.2$ eV²¹. It is important to keep in mind that in the presence of flavor mixing, as is indicated by solar and atmospheric neutrino observations, the bound from beta decays actually applies to the quantity $m_\beta \equiv \sqrt{\sum |V_{ei}|^2 m_i^2}$, since the beta spectrum will actually be an incoherent superposition of spectra (weighted by $|V_{ei}|^2$) with different endpoints, but which are however not resolved by the experimental apparatus. Hence, given the constraints we already have on the mixing angles, and the mass splittings observed, these results already constrain significantly all three neutrino masses.

Regarding the muon neutrino, a direct bound on its mass can be set by looking to its effects on the available energy for the muon in the decay of a pion at rest, $\pi^+ \rightarrow \mu^+ + \nu_\mu$. From the knowledge of the π and μ masses,

and measuring the momentum of the monochromatic muon, one can get the neutrino mass through the relation

$$m_{\nu_\mu}^2 = m_\pi^2 + m_\mu^2 - 2m_\pi \sqrt{p_\mu^2 + m_\mu^2}.$$

The best bounds at present are $m_{\nu_\mu} \leq 170$ keV from PSI, and again they are difficult to improve through this process since the neutrino mass comes from the difference of two large quantities. There is however a proposal to use the muon ($g - 2$) experiment at BNL to become sensitive down to $m_{\nu_\mu} \leq 8$ keV.

Finally, the direct bound on the ν_τ mass is $m_{\nu_\tau} \leq 17$ MeV and comes from the effects it has on the available phase space of the pions in the decay $\tau \rightarrow 5\pi + \nu_\tau$ measured at LEP.

To look for the electron neutrino mass, besides the endpoint of the ordinary beta decay there is another interesting process, but which is however only sensitive to a Majorana (lepton number violating) mass. This is the so called double beta decay. Some nuclei can undergo transitions in which two beta decays take place simultaneously, with the emission of two electrons and two antineutrinos ($2\beta 2\nu$ in fig. 5). These transitions have been observed in a few isotopes (^{82}Se , ^{76}Ge , ^{100}Mo , ^{116}Cd , ^{150}Nd) in which the single beta decay is forbidden, and the associated lifetimes are huge (10^{19} – 10^{24} yr). However, if neutrinos were Majorana particles, the virtual antineutrino emitted in one vertex could flip chirality by a mass insertion and be absorbed in the second vertex as a neutrino, as exemplified in fig. 5 ($2\beta 0\nu$). In this way only two electrons would be emitted and this could be observed as a monochromatic line in the added spectrum of the two electrons. The non observation of this effect has allowed to set the bound $m_{\nu_e}^{Majorana} \equiv |\sum V_{ei}^2 m_i| \leq \text{eV}$ (by the Heidelberg–Moscow collaboration at Gran Sasso). A reanalysis of the results of this experiment even suggest a mass in the range 0.2–0.6 eV, but this controversial claim is expected to be reexplored by the next generation of double beta decay experiments (such as CUORE). There are even projects to improve the sensitivity of $2\beta 0\nu$ down to $m_{\nu_e} \sim 10^{-2}$ eV, and we note that this is quite relevant since as we have seen, if neutrinos are indeed massive, it is somehow theoretically favored (e.g. in the see saw models) that they are Majorana particles.

References

1. E. Fermi, Z. Phys. **88** (1934) 161.
2. H. Bethe and R. Peierls, Nature **133** (1934) 532.
3. F. Reines and C. Cowan, Phys. Rev. **113** (1959) 273.

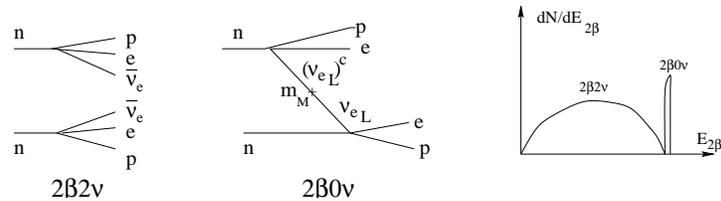


Figure 5. Double beta decay with and without neutrino emission, and qualitative shape of the expected added spectrum of the two electrons.

4. G. Gamow and E. Teller, Phys. Rev. **49** (1936) 895.
5. T. D. Lee and C. N. Yang, Phys. Rev. **104** (1956) 254.
6. C. S. Wu et al., Phys. Rev. **105** (1957) 1413.
7. T. D. Lee and C. N. Yang, Phys. Rev. **105** (1957) 1671; L. D. Landau, Nucl. Phys. **3** (1957) 127; A. Salam, Nuovo Cimento **5** (1957) 299.
8. M. Goldhaber, L. Grodzins and A. W. Sunyar, Phys. Rev. **109** (1958) 1015.
9. R. Feynman and M. Gell-Mann, Phys. Rev. **109** (1958) 193; E. Sudarshan and R. Marshak Phys. Rev. **109** (1958) 1860.
10. G. Feinberg, Phys. Rev. **110** (1958) 1482.
11. G. Danby et al., Phys. Rev. Lett. **9** (1962) 36.
12. E. Majorana, Nuovo Cimento **14** (1937) 170.
13. P. Minkowski, Phys. Lett. **B 67** (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, p. 135, Ed. P. van Nieuwenhuizen and D. Freedman (1979); T. Yanagida, Proc. of the *Workshop on unified theory and baryon number in the universe*, KEK, Japan (1979).
14. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theoret. Phys. **28** (1962) 870.
15. B. Pontecorvo, Sov. Phys. JETP **26** (1968) 984.
16. S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42** (1985) 913; L. Wolfenstein, Phys. Rev. **D17** (1979) 2369.
17. S. N. Ahmed et al. (SNO Collaboration) Phys. Rev. Lett. **92**: 181301 (2004).
18. T. Araki et al. (Kamland Collaboration) hep-ex/0406035.
19. Y. Ashie et al. (Super-Kamiokande Collaboration) Phys. Rev. Lett **93**: 101801 (2004).
20. M. H. Ahn et al. (K2K Collaboration) Phys. Rev. Lett. **90** 041801 (2003).
21. C. Weinheimer et al., hep-ex/0210050; V. Lobashev et al., Nucl. Phys. Proc. Suppl. **91** (2001) 280.