NEUTRINOS

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In these lectures the following topics are considered: historical remarks and general properties, Dirac and Majorana neutrino masses, effective lagrangian approach, the seesaw mechanism, the number of active left-hauded neutrino species, the light neutrino mass matrix, the direct measurement of neutrino masses, double beta decay, neutrino oscillations in vacuum and neutrino oscillations in matter.

1 Introduction.

Recently, important events occured in neutrino physics: the Super-Kamiokande Collaboration reported a strong evidence for neutrino oscillations in their atmospheric neutrino data [1]. The results of the neutrino experiments will be discussed in the Lectures by Dr. L. Di Lella in these Proceedings, including future projects. In the present lecture notes a number of issues pertaining to neutrino physics are considered: general properties, Dirac and Majorama masses, effective lagrangian approach, the seesaw mechanism, the number of active left-handed neutrino species, the light neutrino mass matrix, the direct measurement of neutrino masses, double beta decay, neutrino oscillations in vacuum and neutrino oscillation in matter.

Neutrinos play a very important role in various branches of subatomic physics as well as in astrophysics and cosmology. The neutrino mass problem is the first window to physics beyond the Standard Model of particle physics. The smallness of neutrino mass is likely related to the existence of high mass scales, so high that their direct experimental study is not accessible. The neutrino mass studies may provide some clues to the general problem of fermion mass generation.

It was in 1930 when Wolfgang Pauli wrote his letter addressed to te "Liebe Radioaktive Damen und Herren" (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tübingen. He put forward the hypothesis that, besides electrons and protons, a new particle exists as "constituent of nuclei", the "neutron" ν, able to explain the continuous spectrum of nuclear beta decay $(A, Z) \to (A, Z+1) + e^- + \nu$. The neutrino is light (in Pauli's words: "the mass of the neutron should be of the same order as the electron mass"), neutral (in today's language, neutral in electric charge as well as in colour charge) and has spin 1/2. In 1934, Fermi [2] gave to the neutrino its name and first proposed the four-fermion theory of beta decay, in terms of charged currents. In 1956 the neutrino was observed for the first time by Cowans and Reines [3] in a reactor experiment, by means of its direct detection by its interaction with matter.

2 Neutrino mass and helicity

The first theoretical argument in favour of a vanishing neutrino mass was provided in 1957 by the two-component neutrino theory [4], formulated by Landau, Lee and Yang, and Salam. After the discovery of parity non-conservation by weak interactions, the four-fermion hamiltonian of beta decay had to be written in terms of currents containing both parity even and parity odd components.

$$\mathcal{H} = \sum_i (\bar{p} O^i n)(\bar{e} O_i [G_i + G_i' \gamma_5]\nu) + \text{h.c.} \tag{1}$$

with the Dirac matrices $\mathrm{O}^i = 1, \gamma^\alpha, \sigma^{\alpha\beta}, \gamma^\alpha\gamma_5, \gamma_5$. For any Dirac fermion field ψ, one can write $\psi = \psi_L + \psi_R$, with $\psi_{L,R} = \frac{1 \mp \gamma_5}{2}\psi$ the left and right chiral fields. If the neutrino mass is zero, the chiral projectors $\frac{1 \mp \gamma_5}{2}$ become the helicity projection operators and helicity is Lorentz invariant for zero mass. In the two component neutrino framework, one assumes that only ν_L enters into H. As a consecuence, neutrinos are produced with negative helicity whereas antineutrinos have positive helicity. This picture corresponds to maximal violation of charge conjugation (no left-handed antineutrino) and of parity (no rigth-handed neutrino). The neutrino helicity was measured in 1957 in a cellebrated experiment by Goldhaber et al. [5]: the electron capture reaction

$$\begin{aligned} e^- + {}^{152}Eu \rightarrow \nu + {}^{152}Sm^* \\ \xrightarrow{} {}^{152}Sm + \gamma \end{aligned} \tag{2}$$

leads to a polarized final state which transmits its polarization to the γ-ray of the subsequent decay. The neutrino is thus left-handed, implying $G_i' = -G_i$ in Eq. (1). In this view, the neutrino would be an exceptional particle with $m_\nu = 0$ and the neutrino field is ν_L.

Soon afterwards the V-A theory of weak interactions was proposed [6], in which the left-handed chiral fields of ALL fermions enter into H

$$\mathcal{H} = \frac{G}{\sqrt{2}} 4(\bar{p}_L \gamma^\alpha n_L)(\bar{e}_L \gamma_\alpha \nu_L) + \text{h.c.} \tag{3}$$

For massive particles, one should carefuly distinguish between chirality and helicity. Let us consider the weak decay $\pi^+ \to l^+ + \nu_l$ of pions at rest, with $l = \mu, e$ and ν_l the corresponding neutrino. This semileptonic decay is again described by the product of an hadronic current and a leptonic current. In the V-A structure, the emitted ν_l is left-handed. For $m_\nu = 0$, this also means negative helicity. Conservation of total angular momentum requires l^+ to be of negative helicity too. However, the l^+ are antiparticles and the V-A theory predicts that they are produced with right-handed chirality. As a consequence, the probability amplitude of this process is proportional to the admixture of negative helicity of the charged lepton in its right-handed chirality, i.e., to its mass m_l. Including the phase space factor, one thus expects

$$R_\pi \equiv \frac{\Gamma(\pi^+ \to e^+ + \nu_e)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.28 \times 10^{-4}. \tag{4}$$

When the 4% effect of radiative corrections is considered, this result is in agreement with the experimental value [7] $R_\pi = (1.230 \pm 0.004) \times 10^{-4}$. This manifestation of the chirality suppression follows from the V-A character of the weak current, with a neutrino mass much smaller than that of the charged lepton.

The argument of the two component neutrino theory disappears once it is realized that the left-handed chiral fields are the ones which participate in weak interactions for all fermions. In this sense there is nothing special for neutrinos. There is, however, a crucial difference: except for neutrinos, the right handed chiral fields have to exist in Nature for charged leptons (QED) and for quarks (QED +QCD). Once both left-and right-handed chiral fields are present in particle theory, one has a Dirac mass term for up and down quarks and for (down) charged leptons. In the Standard Model, in which the origin of mass comes from spontaneous symmetry breaking, the Yukawa interaction [8] between the left-handed doublet ψ_L and the right-handed singlet l_R is (in matrix notation in fermion family space)

$$\mathcal{L}_Y^{(\ell)} = -\frac{\sqrt{2}}{v} \bar{\psi}_L M^{(\ell)} \ell_R \varphi + \text{h.c.} \tag{5}$$

with φ the scalar doublet. After spontaneous symmetry breaking,

$$\varphi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}^{(\ell)}_{\text{mass}} = -\bar{\ell}_L M^{(\ell)} \ell_R + \text{h.c.} \tag{6}$$

with $M^{(l)}$ the Dirac mass matrix for charged leptons.

3 Dirac versus Majorana neutrinos

Neutrinos, contrary to other fermions, do not participate in parity conserving vector-like interactions QED or QCD, and only ν_L is relevant for weak interactions. There is no need of introducing ν_R into the theory as an independent field. If one does it, against the "choice" of the Standard Model, the ν_R is sterile against gauge interactions and only suffers the Yukawa interaction

$$\mathcal{L}^{(\nu)}_{Y} = -\frac{\sqrt{2}}{v} \bar{\psi}_L M^{(\nu)} \nu_R \widetilde{\varphi} + \text{h.c.} \tag{7}$$

where $\widetilde{\varphi}$ is the charge-conjugated of the scalar field. The forms (5) and (7) are dictated by $SU(2) \times U(1)$ gauge invariance.

Under spontaneous symmetry-breaking,

$$\widetilde{\varphi} = i\tau_2 \varphi^* \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} \frac{v+H}{\sqrt{2}} \\ 0 \end{array} \right\} \Rightarrow \mathcal{L}^{(\nu)}_{\text{mass}} = -\bar{\nu}_L M^{(\nu)} \nu_R + \text{h.c.} \tag{8}$$

with $M^{(\nu)}$ a Dirac mass matrix for neutrinos. In this alternative, the neutrinos would be Dirac particles, in total analogy with quarks. The leptonic sector would have the analogous to the CKM matrix: the MNS matrix[9] and Eq. (8) induces masses and mixings. The mixing is relevant for charged current interactions, because $M^{(\nu)} \neq M^{(l)}$. The mixing is, however, irrelevant for the neutral current interaction, leading to GIM-suppressed flavour changing neutral currents. Besides Eq. (7), the ν_R's do not appear elsewhere. The matrix $M^{(\nu)}$ leads to lepton flavour violation, but the Lagrangian is still invariant under a Global $U(1)$-Gauge Transformation of Total Lepton Number. Neutrinoless Double Beta Decay would be thus rigorously forbidden in Nature.

As ν_R is sterile for gauge interactions, we can contemplate a second alternative for neutrinos (forbidden for the other fermions): ν_R does not exist as an independent field. We can ask ourselves: Is it possible, with only ν_L, to generate a non-vanishing neutrino mass? A priori, the answer to this question is positive, thanks to Majorana[10]. For the chiral ν_L field, contrary to a Dirac field, its charge conjugated ν_L^c is right-handed, so that one can write a Majorana mass term

$$\mathcal{L}_{\text{mass}}^{(\text{Maj})} = -\frac{1}{2}\bar{\nu}_L M \nu_L^c + \text{h.c.} \tag{9}$$

with only ν_L. For neutrinos, Eq. (9) is not only Lorentz-invariant, but also $SU(3)_{colour} \otimes U(1)_{e.m.}$ invariant. It is thus perfectly legal, contrary to all other fermions carrying conserved charges. The requirement of anticommutation for the quantum fermion fields leads to the symmetry condition $M^T = M$ for the Majorana mass matrix. M is, in general, a complex symmetric matrix and it can be diagonalized by means of a unitary matrix U according to

$$M = UmU^T \tag{10}$$

with m the diagonal matrix of mass eigenvalues. Eq. (9) can be written in terms of the fields χ with definite mass

$$\left.\begin{array}{l} \mathcal{L}_{\text{mass}}^{(\text{Maj})} = -\frac{1}{2}\bar{\chi} m \chi \\ \chi \equiv U^{+}\nu_L + (U^{+}\nu_L)^c \end{array}\right\} \tag{11}$$

As seen, the Majorama field χ is self-conjugated, satisfying

$$\chi = \chi^c \equiv \mathcal{C}\bar{\chi}^T. \tag{12}$$

In this alternative, the physical neutrinos of definite mass would be true neutral particles, with no conserved global lepton number. If a lepton number L is introduced, Eq.(9) transports two units of this lepton charge: $\Delta L = 2$. Neutrinoless Double Beta Decay would be allowed.

In general, the fermion field $\psi(x)$ can be (Fourier) transformed to momentum space by means of the spinor $u_\lambda(p)$ and its charge conjugated $C\left[\overline{u}_\lambda(p)\right]^T$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} \sum_\lambda \left\{ c_\lambda(p) u_\lambda(p) e^{-ipx} + d_\lambda^{+}(p) C[\bar{u}_\lambda(p)]^T e^{ipx} \right\}. \tag{13}$$

where c_λ is the particle annihilation operator and $d^+_\lambda(p)$ is the antiparticle creation operator. With the decomposition (13), one can construct the Dirac propagator. The vacuum expectation value of the time ordered product of the field times its adjoint

$$\langle 0|T\{\psi(x)\overline{\psi}(0)\}|0\rangle \tag{14}$$

is non-vanishing for both Dirac and Majorana neutrinos. It describes neutrino $\rightarrow$ neutrino propagation.

Is there a possibility of Majorana propagation? This is described by the vacuum expectation value of the time ordered product of the field times itself. A non-vanishing value [11] of

$$\langle 0|T\{\psi(x)\psi(0)^T\}|0\rangle \tag{15}$$

is only possible IFF $\psi = \chi = C\ \overline{\chi}^T$, i.e., for a Majorona field. The Majorana condition implies $c_\lambda(p) = d_\lambda(p)$, which identifies particle and antiparticle. The Majorana propagator (15) describes "neutrino$\rightarrow$antineutrino" propagation, a manifestation of the $\Delta L = 2$ character of the Majorana mass term.

In the standard model, the Majorana mass term of Eq.(9) cannot be generated by spontaneous symmetry breaking of a Yukawa interaction with a (doublet) scalar field. The reason is apparent: Eq. (9) is a triplet in weak isospin, so one would need an isotriplet scalar field, which does not exist in the standard model.

As a consequence the standard model, with its particle content and renormalizable dimension-four operators only, predicts that neutrinos are massless.

4 Effective Lagrangian Approach

In the last 30 years one has seen a deep evolution in the understanding of quantum field theories, so that the requirement of renormalizability is taken today with a less dogmatic philosophy. Take the particle content of the standard model and ask what is the lowest dimension non-renormalizable operator which still keeps the $SU(2) \times U(1)$ gauge invariance. The answer to this question is unique, given by the dimension-five operator [12] in the leptonic sector

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2\Lambda}(\bar{\tilde{\psi}}_L\varphi)F(\psi\widetilde{\varphi}^+\psi_L), \tag{16}$$

where $\widetilde{\psi} = i\tau_2\psi^c = i\tau_2 C\overline{\psi}^T$. We can say properly that (16) represents the first window to physics beyond the standard model. The symmetric $F = F^T$ matrix induces mixing in flavour-space. The coupling $\frac{1}{\Lambda}$ is reminiscent of the scale of new physics at higher energies. Eq. (16) generates, in addition to lepton flavour violation, $\Delta L = 2$ transitions.

After spontaneous symmetry breaking, Eq. (16) leads to a Majorana mass term (9) for neutrinos, with the mass matrix

$$M = \frac{v^2}{\Lambda} F. \qquad (17)$$

We conclude that this new physics mechanism leads to massive Majorana neutrinos. Eq. (17) provides a very simple and attractive explanation of the smallness of neutrino mass. It relates the smallness of $m_\nu$with the existence of a very large mass scale Λ compared with the electroweak scale, given by the vacuum expectation value $v = 174\ GeV$. The effective Lagrangian approach cannot unveil the origin of Λ, because the shorter-distance physics has been integrated out. We will discuss below the see-saw mechanism based on the introduction of very heavy right-handed ν_R. An alternative to it is suggested by the Fierz-reordered form of the effective lagrangian(16)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4\Lambda}(\bar{\widetilde{\psi}}_L F \vec{\tau}\psi_L)(\widetilde{\varphi}^+ \vec{\tau}\varphi). \qquad (18)$$

The last bracket has the same transformation property of a scalar triplet, so that a heavy Higgs triplet will do the job as well. This theory is not very attractive nowadays, and it runs into difficulties with the invisible Z-width.

Several models of neutrino mass and mixing based on the minimalist approach of this Section are discussed in Ref. [13].

5 Seesaw mechanism

The seesaw mechanism [14] is most natural in the framework of grand unified theories, such as $SO(10)$, or left-right symmetric models, in which the rigth-handed ν_R acquires a large Majorana mass (Λ) as part of the symmetry breaking scenarios. But it also operates in the standard $SU(2) \times U(1)$ gauge invariant model, extended to include a heavy ν_R.

The most general mass term includes not only the left-right Dirac mass, generated by the standard spontaneous symmetry breaking in the Yukawa coupling, but also a Majorana mass for right-handed neutrinos. As ν_R is sterile (electroweak singlet), this last term introduced by hand keeps the $SU(2) \times U(1)$ gauge invariance. We can write

$$\mathcal{L}_{\text{mass}}^{D-M} = -\bar{\nu}_R m_D \nu_L - \frac{1}{2}\bar{\nu}_R m_R (\nu_R)^c + \text{h.c.} = -\frac{1}{2}(\bar{n}_L)^c M n_L + \text{h.c.}, \qquad (19)$$

where the last equality follows from organizing the left handed fields (twice the number of families) as

$$n_L \equiv \begin{Bmatrix} \nu_L \\ (\nu_R)^c \end{Bmatrix}, \quad M = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}. \qquad (20)$$

Let us discuss the limit $m_D << m_R$ for the one-family case, where M is a 2×2 real symmetric matrix. The diagonalization of M leads to two Majorana neutrinos with masses

$$m_1 \approx \frac{m_D^2}{m_R}, \quad m_2 \approx m_R, \qquad (21)$$

and definite CP eigenvalues $\eta_1 = -1$, $\eta_2 = 1$. The mixing angle is hierarchical $\theta \simeq m_D/m_R$. Neglecting this small admixture between "active" and "sterile" neutrinos, the Majorana Fields with definite mass are

$$\nu_1 \approx i\nu_L - i(\nu_L)^c; \quad \nu_2 = \nu_R + (\nu_R)^c, \qquad (22)$$

corresponding to a light neutrino with $m_1 << m_D$ and a heavy neutrino with $m_2 >> m_D$. This solution is a realization of the discussion of the previous Section, with the high mass scales Λ represented by m_R.

The mass Lagrangian of Eq.(19) violates global lepton number L only by the right-handed Majorana term -$\frac{1}{2}\overline{\nu}_R m_R (\nu_R)^c$, characterized by the large mass. One thus connects the smallnes of the light neutrino mass to lepton number violation at the high mass scale.

The results for one family can be generalized, so that m_D and m_R are matrices of dimension the number of families. Block-diagonalization of M gives

$$m_L \approx -m_D m_R^{-1} m_D^T; \quad m_R, \tag{23}$$

for the active and sterile neutrinos, respectively. The subsequent diagonalization of m_L leads to active Majorana neutrinos with an expected hierarchy of masses $m_1 << m_2 << m_3$, when taking into account that m_D is of the order of quark or lepton masses.

6 How many active neutrinos?

The counting of light active left-handed neutrinos is based on the family structure of the standard model and assuming its predictions of a universal diagonal neutral current coupling. We have the vertex of Fig.1

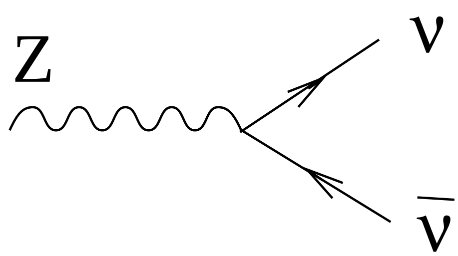

Figure 1.

$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}. \tag{24}$$

With this current and the condition $m_{\nu_i} < m_Z/2$ for the physical neutrinos, one can use the total width $\Gamma_Z$of the Z-boson to extract N_ν. A "nearly" model independent method can be put forward as follows

$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} \equiv \frac{1}{\Gamma_\nu}(\Gamma_Z - \Gamma_h - 3\Gamma_\ell) = \frac{\Gamma_\ell}{\Gamma_\nu}\left[\sqrt{\frac{12\pi R_{h\ell}}{\sigma_h^0 m_Z^2}} - R_{h\ell} - 3\right], \tag{25}$$

where $\Gamma_{inv}$is the invisible width, Γ_h the total hadronic width and Γ_l one of the charged lepton widths. The last equality leads to a bracket with only experimental inputs: the hadronic cross section σ_h^0 at the peak of mass m_Z

and the hadronic to leptonic ratio R_{hl} of widths. The factor in front has to be taken from the standard model, but the ratio Γ_l/Γ_ν is free from big universal radiative corrections. The precise maping of the Z line shape from the four experiments of LEP1 facility shows a clear demonstration of $N_\nu = 3$. Using the strategy of Eq.(25), one obtains [7]

$$N_\nu = 2.994 \pm 0.012. \tag{26}$$

7 The light neutrino mass matrix

In the three-flavour framework, without light sterile neutrinos, the relevant mixing matrix is that indicated by m_L in Eq.(23), whatever its origin is. In a hierarchical solution for the three neutrino masses, $\Delta m^2_{32} \simeq \Delta m^2_{31}$could be identified with the mass difference relevant to neutrino oscillations in the atmospheric neutrino data, whereas $\Delta m^2_{21} << \Delta m^2_{32}$ would be associated with the solar neutrino problem. The two-flavour analysis is, in fact, a good first approximation to the results of the three-flavour studies [15] because the mixing angle θ_{13} is constrained to be rather small by the CHOOZ data [16] $|U_{e3}|^2 < 0.02$.

With the assumptions of this non-participation of ν_e in atmospheric oscillations plus a maximal atmospheric neutrino mixing, compatible with the experiment [1], we can construct the real and orthogonal diagonalizing matrix U as

$$\begin{aligned} U &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \end{aligned} \tag{27}$$

As a result of $U_{e3} = 0$, the theory is CP-symmetric and U can be chosen real. It has been constructed as a rotation in the (12)-plane and then a rotation by $\pi/4$ in the (23)-plane. The factorization of atmospheric and solar mixings will be lost once $U_{e3} \neq 0$ is allowed. This in turn induces CP violation in the leptonic sector, through a phase that could not be rotated away. The determination of these parameters is the objective of the long term neutrino projects.

Solar neutrino experiments still allow several solutions for the rotation in the (12)-plane. Once determined, this information plus the neutrino spectrum provide an empirical mass matrix from

$$m_L = U \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U^T$$

8 Direct measurement of neutrino mass

Neutrino oscillations (see below) constitute the most precise method to search and measure neutrino mass differences. In order to determine the absolute mass scale, one needs other observables. Fermi proposed [2] a kinematic search of neutrino mass from the hard part of the beta spectra in ^3H beta decay. With some abuse of language, this search has produced an upper limit to the electron neutrino mass. The electron neutrino is a weak state. Due to mixing, it has no definite mass. For the mixing of Eq.(27), the discussion of this section applies to ν_1 with probability c^2 and to ν_2 with probability s^2.

For "allowed" nuclear trnsitions, the nuclear matrix elements do not generate any energy dependence, so that the electron spectrum is given by phase space alone

$$\frac{dN}{dT} = CpE(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}F(E), \tag{28}$$

where $E = T + m_e$, Q is the maximum energy and $F(E)$ the Fermi function which incorporates final state Coulomb interactions.

The "classical" decay $^3H \to ^3 He + e^- + \overline{\nu}_e$ is a superallowed transition with a very small energy release $Q = 18.6\ KeV$. In the Kurie plot

$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{pEF(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}}, \tag{29}$$

a non-vanishing neutrino mass m_ν provokes a distorsion from the straight-line T-dependence at the end point of the spectrum, in such a way that $m_\nu = 0 \to T_{\max} = Q$ whereas $m_\nu \neq 0 \to T_{\max} = Q - m_\nu$. This is shown in Fig.2

The experimental spectrum is fitted by m_ν^2 and many other parameters (Q, background term, normalization, ...). The most precise Troitsk and Mainz experiments [7] give no indication in favour of $m_\nu \neq 0$. One has the upper limit $m_\nu < 2.5\ eV$ at 95% c.l.

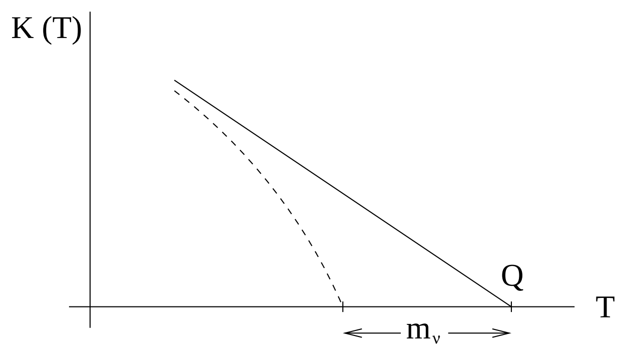

Figure 2.

9 Neutrinoless Double Beta Decay

One of the main pending questions in neutrino physics is: Are neutrinos Dirac or Majorana particles? Or, equivalently: Is the neutrino its own antiparticle? The identity of ν and $\overline{\nu}$ would mean that both ν and $\overline{\nu}$ interact with matter in the same way. Do they? A simple inspection to the experimental cross section [7] in the region of energies in which the behaviour is linear with the energy in the lab shows that there is a difference: the neutrino cross section is about twice the result for antineutrinos. This comparison is, however, not relevant for our question, because the difference in the total cross section for neutrinos and antineutrinos is due merely to the different polarizations of the beams. The so-called neutrinos are left-handed whereas the so-called antineutrinos are right-handed and the cross section is helicity dependent.

Contrary to the situation described in the total cross section, a test of the Majorana condition $c_\lambda(p) = d_\lambda(p)$ (in Eq.(13)) needs the preparation of neutrinos and antineutrinos of the same helicity.

The way to study this problem is based on the $\Delta L = 2$ transition implied by the Majorana mass term. The neutrinoless double beta decay process

$$(A,\, Z) \rightarrow (A,\, Z+2) + e^- + e^- \tag{30}$$

was proposed by Furry in 1939 [15] and becomes allowed for Majorana neutrino virtual propagation. It is described by the diagram of Fig.3

as a second order weak interaction amplitude. It becomes a source of nuclear instability for selected even-even nuclei in which the single beta decay is energetically forbidden. I show the level diagram corresponding to the decay

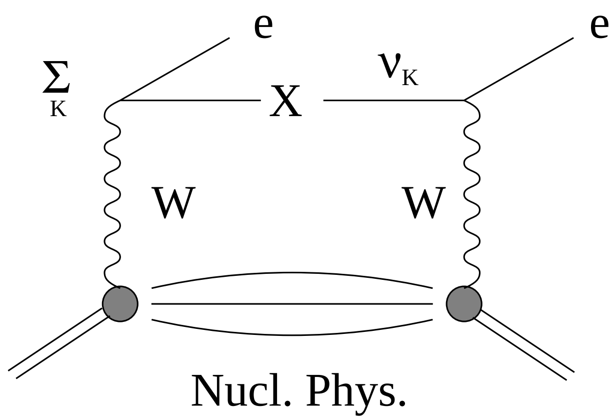

Figure 3.

of ^{76}Ge in Fig.4

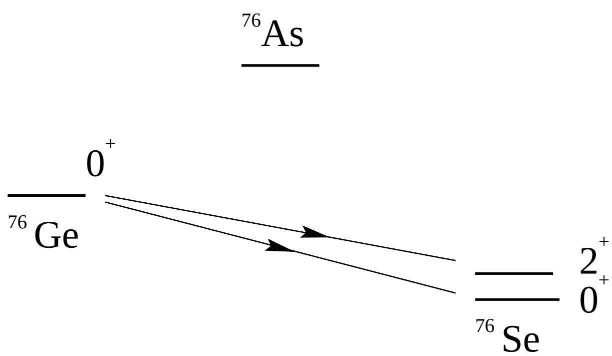

Figure 4.

The neutrino propagator is here

$$\widehat{\nu_{eL}(x_1)\nu}^T_{eL}(x_2) = -\sum_k U^2_{ek}\frac{1-\gamma_5}{2}\widehat{\chi_k(x_1)}\bar{\chi}_k(x_2)\frac{1-\gamma_5}{2}C$$
$$= \sum_k U^2_{ek} m_k \frac{-i}{(2\pi)^4}\int d^4p \frac{e^{ip(x_1-x_2)}}{p^2-m_k^2}\frac{1-\gamma_5}{2}C. \qquad (31)$$

If m_k are small, compared to the momenta relevant for nuclear physics excitations, the neutrino masses can be neglected in the denominator of the propagator. The amplitude of the process is then factorized in its different ingredients

$$\mathrm{Amp}[\beta\beta_{0\nu}] = \langle m_\nu \rangle (\text{Phase Space})(\text{Nuclear Physics}). \tag{32}$$

The quantity of primary interest in neutrino physics is the average neutrino mass $< m_\nu >$

$$\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k. \tag{33}$$

This result shows that the main ingredient to produce an allowed $(\beta\beta)_{0\nu}$ is the massive Majorana neutrino character. Even without mixing, the process is still allowed. In presence of mixing, there are contributions of the different physical neutrinos to $< m_\nu >$, contributions which can cancel each other. Even with CP-conserving interactions, the contributions of different CP eigenvalues η_k appear as

$$\langle m_\nu \rangle = \sum_k |U_{ek}|^2 m_k \eta_k. \tag{34}$$

Besides these properties, the result (32) shows the dependence of the amplitude with the absolute neutrino masses, not with the mass differences. Under favourable circunstances, a positive signal of the $(\beta\beta)_{0\nu}$ process could be combined with results of neutrino oscillation studies to determine [18] the absolute scale of neutrino masses.

The most stringent experimental limit on $< m >$ at present is obtained by the Heidelberg-Moscow collaboration [19], $< m >< 0.35 eV$, running in the Gran Sasso underground laboratory. There are prospects to reach sensitivities of $10^{-2} eV$ in the near future.

To compare with, I should add that the two-neutrino double beta decay

$$(A,\, Z) \rightarrow (A,\, Z+2) + (2e^-) + (2\bar{\nu}_e) \tag{35}$$

is allowed, although rare, even if global lepton number is conserved in second order weak interaction decays as shown in Fig.5

Direct counter experiments have observed $(2\beta)_{2\nu}$ in a variety of nuclei, with half-lifes of the order $10^{19} - 10^{24}$ years.

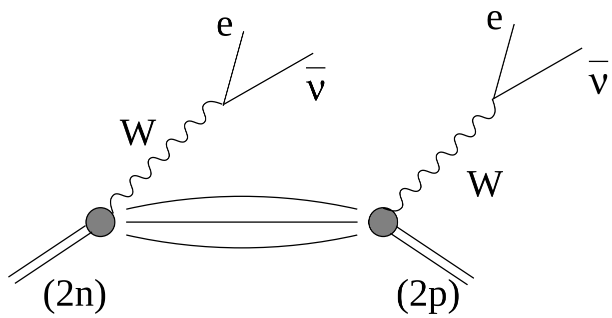

Figure 5.

10 Neutrino Oscillations

The most sensitive method to prove that neutrinos are massive is provided by neutrino oscillations [20]. Neutrino oscillations are quantum mechanical processes based on mass and mixing of the neutrino flavours. If the weak interaction states (greek indices) do not coincide with the mass eigenstates (latin indices), the first ones are coherent superpositions of definite mass states

$$\nu_\alpha = \sum_k U_{\alpha k}\nu_k, \tag{36}$$

where ν_k can be either Dirac or Majorana particles. For the present discussion, we will limit ourselves to active left-handed ν_k's, although the superposition (36) has to be extended to light sterile neutrinos if they exist in Nature.

The propagation of the state ν_α in vacuum, if it was prepared as such at $t = 0$, is

$$|\nu_\alpha(t)\rangle = \sum_\beta |\nu_\beta\rangle \left(\sum_k U^*_{\beta k} e^{-iE_k t} U_{\alpha k} \right). \tag{37}$$

The transition probability that (37) be observed, at a distance $L \simeq t$, as ν_β is given by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\beta\alpha} + 2\mathrm{Re} \sum_{k<j} U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j} \left(e^{-i\Delta m^2_{kj}\frac{L}{p}} - 1 \right), \tag{38}$$

where $p \simeq E$ is the neutrino momentum ($\simeq$ *energy*) and Δm^2_{kj} the square mass differences of the physical neutrinos.

One realizes that the conditions

$$\Delta m^2_{kj} \frac{L}{E} << 1, \quad \forall k \neq j, \tag{39}$$

if satisfied for all neutrinos, lead immediately to $P(\nu_\alpha \to \nu_\beta) \simeq \delta_{\beta\alpha}$. We conclude that, in order to observe neutrino oscillations, in addition to mixing one needs at least one $\Delta\ m^2$ with $\Delta\ m^2 \gtrsim \frac{E}{L}$. The "canonical" sensitivity of some natural and person-made neutrino sources is given in the Table

	Sun	Atmosph	Reactors	Meson Factories	H.E. Accel
$\frac{E}{L}(eV)^2$	10^{-11}	10^{-3}	10^{-2}	10^{-1}	1

These values can however be modified by either long-base-line experiments with intense neutrino beams or the effect of neutrino interactions in matter (see next Section).

The flavour detection at distance L by means of charged current interactions allows the classification of neutrino oscillation experiments into two types:

i) Appearance Experiments, described in Fig.6

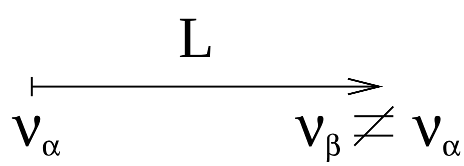

Figure 6.

ii) Disappearance Experiments, in which one measures the Survival Probability, as shown in Fig.7

For low energy $\overline{\nu}_e(\nu_e)$ neutrinos such as produced by reactors (the sun), one is automatically limited to disappearance experiments. Pure neutral current neutrino detection does not discriminate among flavours, so that it is insensitive to neutrino oscillations. The neutrino detection by elastic scattering on electrons is, in general, dominated by charged current interactions with some proportion of neutral current scattering. An interesting exception

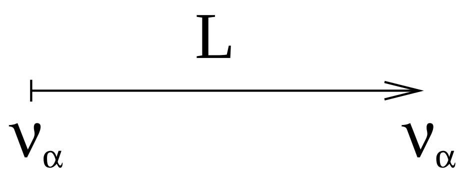

Figure 7.

to this rule happens for reactor antineutrinos $\overline{\nu}_e$ at energies around 0.5 MeV, where a dynamical zero [21] operates for $(\overline{\nu}_e\ e)$ scattering only, but not for $(\overline{\nu}_\mu\ e)$ or $(\overline{\nu}_\tau e)$ scattering. The neutrino oscillation is then manifested [22] like an appearance experiment. By varying the energy of the $\overline{\nu}_e$, one could tune the proportion of appearance versus disappearance behaviours.

For an oscillation between two neutrino types, the mixing matrix of Eq.(36) is real and orthogonal

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{40}$$

so that the appearance and survival probabilities are given, correspondingly, by $(\nu' \neq \nu)$

$$\left.\begin{array}{l} P(\nu \to \nu') = \frac{1}{2}\sin^2 2\theta \left(1 - \cos\frac{\Delta m^2 L}{2E}\right) \\ P(\nu \to \nu) = 1 - P(\nu \to \nu') \end{array}\right\}. \tag{41}$$

With two intervening parameters $(\Delta m^2, \sin^2 2\theta)$, the analysis of neutrino oscillation experiments is presented in “exclusion” plots.

The general mixing for three families contains three angles and one CP phase, accompanying two independent mass differences. All these ingredients have to have an active participation in order to generate CP violating observables [23]. A program of this kind needs intense beams of high energy neutrinos with different flavours and well known spectra.

For a hierarchical spectrum of neutrinos $m_1 << m_2 << m_3$, we can assume $\Delta m^2_{12}\frac{L}{2E} << 1$, except for solar or cosmic neutrinos. Under the assumption of a single relevant $\Delta m^2_{23} \simeq \Delta m^2_{13}$mass difference, the appearance probability in the three-family case $(\beta \neq \alpha)$ becomes

$$P_{\alpha\to\beta} = 2|U_{\beta 3}|^2|U_{\alpha 3}|^2\left(1-\cos\frac{\Delta m_{23}^2 L}{2E}\right), \qquad (42)$$

which has the same oscillating form as for the case of two neutrino types. The effective "mixing" has however a different meaning. The survival probability is given by

$$P_{\alpha\to\alpha} = 1-\sum_{\beta\neq\alpha}P_{\alpha\to\beta} = 1-2|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\left(1-\cos\frac{\Delta m_{23}^2 L}{2E}\right). \qquad (43)$$

As a consequence, the Disappearance Reactor Experiments (CHOOZ, Palo Verde) are primarily a measure of $|U_{e3}|$.

11 Neutrino oscillations in matter

In a medium, the electron-neutrinos ν_e acquire an extra inertia due to the extra charged current interaction with the electrons of matter, described by the first diagram of Fig.8

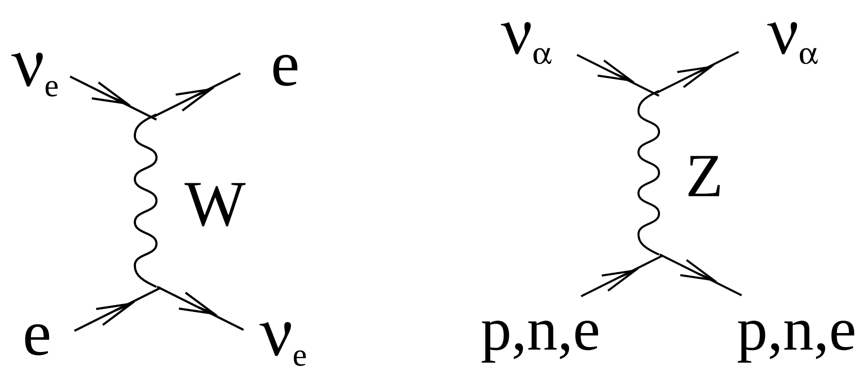

Figure 8.

As a consecuence, in their propagation the ν_e acquires an extra phase coming from $< H_{cc} >$. The vector charge density for electrons contributes coherently to forward scattering, so that $<\overline{e}\ \gamma^0\ e>=<e^+\ e>=n_e$, with n_e the electron number density. The other terms of $H_{cc}$are not coherent, so that

$$\langle H_{cc} \rangle \approx \sqrt{2} G_F n_e \tag{44}$$

for ν_e. All flavours $(\nu_e, \nu_\mu, \nu_\tau)$ have a common neutral current interaction, described by the second diagram of Fig.8, which leads to a common phase in their propagation. This common phase is irrelevant.

In the flavour basis, the matrix elements of the Hamiltonian in vacuum are given by $U_v \; H_v \; U_v^+$, where $(H_v)_{ij} = (p + \frac{m_i^2}{2E})\delta_{ij}$ and U_v is the mixing matrix. For two families, it is given by Eq.(40) and the effective evolution equation, up to terms proportional to identity, is

$$i\frac{d}{dt}\left\{ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right\} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta_v & \frac{\Delta m^2}{4E}\sin 2\theta_v \\ \frac{\Delta m^2}{4E}\sin 2\theta_v & \frac{\Delta m^2}{4E}\cos 2\theta_v \end{pmatrix} \left\{ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right\}. \tag{45}$$

In matter, the effective hamiltonian in the flavour basis is obtained from Eq. (45) by the addition of the diagonal non-universal charged current matrix element (44). Once again, up to terms proportional to the identity (as it is the case for the neutral current interaction), the evolution in matter is governed by

$$i\frac{d}{dt}\left\{ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right\} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta_v + \sqrt{2} G_F n_e & \frac{\Delta m^2}{4E}\sin 2\theta_v \\ \frac{\Delta m^2}{4E}\sin 2\theta_v & \frac{\Delta m^2}{4E}\cos 2\theta_v \end{pmatrix} \left\{ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right\}. \tag{46}$$

The diagonalization of the effective hamiltonian of Eq.(46) by means of

$$U_M = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix} \tag{47}$$

leads to the following solution for θ_M, at constant density n_e,

$$\tan 2\theta_M = \frac{\tan 2\theta_v}{1 - \frac{L_v}{L_e}\frac{1}{\cos 2\theta_v}}, \tag{48}$$

where L_e is the interaction length of ν_e and L_v is the oscillation length in vacuum:

$$L_e = \frac{\sqrt{2}\pi}{G_F n_e}; \quad L_v = \frac{4\pi E}{\Delta m^2}. \tag{49}$$

The matter stationary eigenstates do not coincide with the mass eigenstates in vacuum. It is remarkable from Eq.(48) that, independent of how small θ_v could be, θ_M can give maximal mixing ($= \frac{\pi}{4}$) if a "resonance condition" is satisfied as given by

$$\left(\frac{L_v}{L_e}\right)_{\text{res}} = \cos 2\theta_v. \tag{50}$$

This resonance enhancement constitutes the cellebrated MSW effect [24]. In view of Eqs.(49), the resonance condition appears at a "resonance energy" for n_e and Δm^2 fixed. For the resonance enhancement to be possible, $(\Delta m^2 \cos 2\ \theta_v)$ has to be positive for neutrinos. With the natural convention (1,2) for the order of the mass eigenvalues, $\Delta m^2 > 0$, so that Eq.(50) needs

$$\cos 2\theta_v = \cos^2\theta_v - \sin^2\theta_v > 0, \tag{51}$$

i.e., the lowest mass eigenstate has to have a larger ν_e component. In the case of antineutrinos, the coherent interaction amplitude (44) changes sign, so that the resonance condition is only possible if $\cos 2\ \theta_v < 0$. We conclude that either neutrinos or antineutrinos, but not both, can show the resonance enhancement.

The appearance probability for neutrinos in matter is dictated by the matter mixing angle θ_M and the energy gap $E_2^M - E_1^M$ in matter. One obtains

$$|\langle\nu_\mu|\nu_e(t)\rangle|^2 = \sin^2 2\theta_M \sin^2 \frac{\pi L}{L_M(E)}, \tag{52}$$

where $L_M(E)$ is the energy dependent "matter oscillation length"

$$L_M(E) = \frac{L_v}{\left[1 - 2\frac{L_v}{L_e}\cos 2\theta_v + \left(\frac{L_v}{L_e}\right)^2\right]^{1/2}}. \tag{53}$$

Two comments in connection with the oscillation probability (52):

(i) The probability of mixing

$$\sin^2 2\theta_M = \frac{\left(\frac{\sin 2\theta_v}{L_v}\right)^2}{\left(\frac{\cos 2\theta_v}{L_v} - \frac{1}{L_e}\right)^2 + \left(\frac{\sin 2\theta_v}{L_v}\right)^2}, \tag{54}$$

has a typical resonance form, with the maximun (= 1) at the resonance energy (50).

(ii) The oscillation phase does not depend explicitly on $\frac{L}{E}$ anymore, but the result (52) is still an even function of L.

When the matter has a varying density, like the case of neutrinos propagating from the center of the sun, the evolution equation(46) cannot be solved analitically, except for a few selected functional dependences. There is an important case, however, in which one can discuss a simple approximate solution: that of an adiabatically, slowly, varying density. Let us consider electron neutrinos generated in a region of high density, like the center of the sun, much higher that the value associated with the resonance condition. Eq.(48) tells us that the mixing angle in matter is $\theta_M \simeq \frac{\pi}{2}$, i.e., ν_e is near the higher energy state in matter and neutrino mixing is suppressed. As neutrinos propagate towards regions of smaller density, θ_M decreases and eventually reaches the resonance condition, in which neutrino mixing is maximal $\theta_M = \frac{\pi}{4}$. Further propagation to smaller densities, like for neutrinos leaving the sun, leads to level crossing and $\theta_M \to \theta_v$. If this is small, ν_e is near the lowest energy state and neutrino mixing is again small. For adiabatic evolution, the system remains in the corresponding stationary state in level ordering. If originally the electron neutrino was in the upper level, the system will evolve to the upper level of the modified hamiltonian, which at the end is near the muon neutrino. We conclude that there is complete Flavour Conversion in this particular case. This discussion is illustrated in Fig.9, which shows the energy levels as a function of the matter density

12 Outlook

The problem of neutrino masses and mixing has entered a very fascinating era. More than fourty experiments are devoted to the field and the prospects for positive results and definite answers look accessible.

Massive neutrinos can open a unique window to a new scale in physics. We still do not know whether neutrinos are Dirac or Majorana particles.

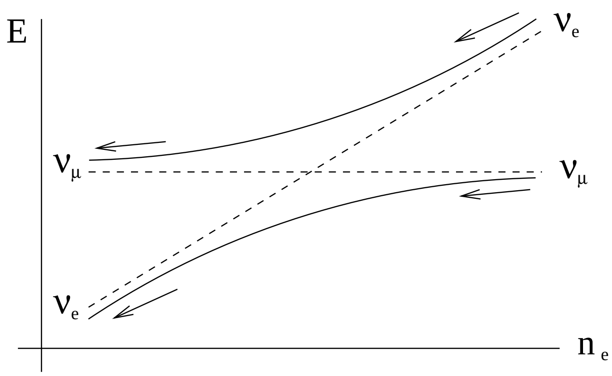

Figure 9.

Similarities and differences with the quark mixing can provide clues to the flavour problem.

There are several subjects related to neutrino physics not covered in these lectures. Among them, one can cite CP, and T, violation in the leptonic sector, the connection to flavour changing neutral current processes, electric and magnetic dipole moments, high energy neutrino astronomy. Some of these topics can be followed in the excellent lectures by Akhmedov[25]. The experimental status of neutrino oscillation studies is discussed by Di Lella in these Proceedings.

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