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Theories of Baryogenesis

Antonio Riotto *,[1]

* *CERN, Theory Division,*
CH-1211, Geneva 23, Switzerland.
E-mail: riotto@nxth04.cern.ch

Abstract

These lectures provide a pedagogical review of the present status of theories explaining the observed baryon asymmetry of the Universe. Particular emphasis is given on GUT baryogenesis and electroweak baryogenesis. The key issues, the unresolved problems and the very recent developments, such as GUT baryogenesis during preheating, are explained. Some exercises (and their solution) are also provided.

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[1]On leave of absence from Theoretical Physics Department, University of Oxford,U.K.

Contents

1 Introduction

We do not know the history of the observable Universe before the epoch of nucleosynthesis, but it is widely believed that there was an early era of cosmological inflation. The attraction of this paradigm is that it can set the initial conditions for the subsequent hot big-bang, which otherwise have to be imposed by hand. One of these is that there be no unwanted relics (particles or topological defects which survive to the present and contradict observation). Another is that the initial density parameter should have the value $\Omega = 1$ to very high accuracy, to ensure that its present value has at least roughly this value. There is also the requirement that the Universe be homogeneous and isotropic to high accuracy. The flatness and the horizon problems of the standard big bang cosmology are – indeed – elegantly solved if during the evolution of the early Universe the energy density happens to be dominated by the vacuum energy of a scalar field – the inflaton – and comoving scales grow quasi-exponentially.

At the end of inflation the energy density of the Universe is locked up in a combination of kinetic energy and potential energy of the inflaton field, with the bulk of the inflaton energy density in the zero-momentum mode of the field. Thus, the Universe at the end of inflation is in a cold, low-entropy state with few degrees of freedom, very much unlike the present hot, high-entropy universe. The process by which the inflaton energy density is converted into radiation is known as reheating. What is crucial about these considerations is that, at the end of inflation, the Universe does not contain any matter and – even more important – the Universe looks perfectly baryon symmetric – there is no dominance of matter over antimatter.

The observed Universe – however – is drastically different. We do not observe any bodies of antimatter around us within the solar system and if domains of antimatter exist in the Universe, they are separated from us on scales certainly larger than the Virgo cluster (~ 10 Mpc). The Universe looks baryon asymmetric to us. Considerations about how the light element abundances were formed when the Universe was about 1 MeV hot lead us to conclude that the difference between the number density of baryons and that of antibaryons is about 10^{-10} if normalized to the entropy density of the Universe.

Theories that explain how to produce such a tiny number go generically under the name of *Theories of baryogenesis* and they represent perhaps the best example of the perfect interplay between particle physics and cosmology. Until now, many mechanisms for the generation of the baryon asymmetry have been proposed and we have no idea which is the correct one. Grand Unified Theories (GUTs) unify the strong and the electroweak interactions and predict baryon number violation at the tree level. They are – therefore – perfect candidates for a theory of baryogenesis. There, the out-of-equilibrium decay of superheavy particles can explain the observed baryon asymmetry, even though there remain problems strictly related to the dynamics of reheating after inflation. In the theory of electroweak baryogenesis, baryon number violation takes place at the quantum level due to the chiral anomaly. Baryogenesis scenarios at the electroweak scale have been the subject of intense activity in the last few years. They are certainly attractive because they can be tested at the current and future accelerator experiments.

The bottom line of all this intense research is that, within the standard model of weak interactions, it is difficult, if not impossible, to explain how the generation of the baryon asymmetry took place. Therefore, the observation of a baryon asymmetry in the Universe is an indication that the description of Nature cannot be limited to the Weinberg-Salam theory, something else is called for.

The goal of these lectures is to provide a pedagogical review of the present state of baryogenesis, with particular emphasis on GUT baryogenesis and electroweak baryogenesis. The technical details of the numerous models considered in the literature are not elaborated, but the key points, the unresolved problems and the very recent developments – such as GUT baryogenesis during preheating – are presented. We hope that this approach will help the reader to get interested in this fascinating subject. A different focus may be found in other accounts of the subject [29, 113, 122, 38]. Some exercises (and their solution) are also provided.

The review is laid out as follows. Section 2 describes some necessary tools of equilibrium thermodynamics. Section 3 contains some considerations about the baryon symmetric Universe and explains the three basic conditions necessary to generate the baryon asymmetry. The standard out-of-equilibrium scenario is addressed in Section 4, while Section 5 contains informations about GUT baryogenesis and the thermal history of the Universe, with particular attention paid to the recent developments related to the theory of preheating. Section 6 is dedicated to the issue of baryon number violation in the standard model and its possible implications for GUT baryogenesis and leptogenesis. Finally, section 7 addresses the rapidly moving subject of electroweak baryogenesis.

A note about conventions. We employ units such that $\hbar = c = k = 1$ and references are listed in alphabetic order.

2 Some necessary notions of equilibrium thermodynamics

2.1 Expansion rate, number density, and entropy

Before launching ourselves into the issue of baryon asymmetry production in the early Universe, let us just remind the reader a few notions about thermodynamics in an expanding Universe that will turn out to be useful in the following. According to general relativity, the space-time evolution is determined via the Einstein equation by the matter content of the Universe, which differs from epoch to epoch depending on what kind of energy dominates the energy density of the Universe at that time. There are three important epochs characterized by different relation between the energy density ρ and the pressure p: *1)* vacuum energy dominance with $p = -\rho$, *2)* massless (relativistic) particle dominance with $p = \rho/3$ and *3)* nonrelativistic particle dominance with $p \ll \rho$. The Einstein equation reads

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_N T_{\mu\nu}, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor, $\mathcal{R}$ is the Ricci scalar, $g_{\mu\nu}$ is the metric, $G_N = M_{\rm P}^{-2} = (1.2 \times 10^{19})^{-2}\ {\rm GeV}^{-2}$ is the Newton constant and $T_{\mu\nu}$ is the stress-energy tensor.

With the homogeneity and isotropy of the three-space, the Einstein equation is much simplified with the Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\vec{x}^2, \tag{2}$$

where $a(t)$ is the cosmic scale factor and the stress-energy tensor is reduced to $T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho) u_\mu u_\nu$. Here u^μ is the velocity vector which in the rest frame of the plasma reads $u^\mu = (1, \mathbf{0})$ and has the property $u^\mu u_\mu = 1$. The $0-0$ component of eq. (1) becomes the so-called Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_N}{3}\rho, \tag{3}$$

where k can be chosen to be $+1$, -1 or 0 for spaces of constant positive, negative or zero spatial curvature, respectively, and we have defined the Hubble parameter

$$H \equiv \frac{\dot{a}}{a}, \tag{4}$$

which meaures how fast the Universe is expanding during the different stages of its evolution.

The $\mu = 0$ component of the conservation of the stress-energy tensor ($T^{\mu\nu}_{;\nu} = 0$) gives the first law of thermodynamics in the familiar form

$$d(\rho a^3) = -p\, d(a^3), \tag{5}$$

that is, the change in energy in a comiving volume element, $d(\rho a^3)$ is equal to minus the pressure times the change in volume, $p\, d(a^3)$. For a simple equation of state $p = w\rho$, where w is independent of time, the energy density evolves like $\rho \propto a^{-3(1+w)}$. Examples of interest include radiation ($\rho \propto a^{-4}$), matter ($\rho \propto a^{-3}$), vacuum energy ($\rho \propto$ constant). The time-behaviour of the scale factor $a(t)$ then is

$$\begin{aligned} &\mathit{1)}\ a \propto e^{Ht},\ H = \sqrt{\frac{8\pi V}{3M_{\rm P}^2}}, \\ &\mathit{2)}\ a \propto t^{1/2}, \\ &\mathit{3)}\ a \propto t^{2/3}. \end{aligned} \tag{6}$$

The first stage is the inflationary epoch where the constant vacuum energy V gives the exponential growth of the scale factor, which is believed to solve the horizon and the flatness problems of the standard big-bang theory of cosmology [54] (for a review, see [86]). Of great importance is the transient stage from inflation to radiation dominance. This epoch is called reheating after inflation and we shall come back to it later in these lectures.

What is relevant for us is that the early Universe was to a good approximation in thermal equilibrium at temperature T [73] and we can define the equilibrium number density $n_X^{\rm EQ}$ of a generic interacting species X as

$$n_X^{\rm EQ} = \frac{g_X}{(2\pi)^3} \int f_{\rm EQ}(\mathbf{p}, \mu_X)\, d^3p, \tag{7}$$

where g_X denotes the number of degrees of freedom of the species X and the phase space occupancy $f_{\rm EQ}$ is given by the familiar Fermi-Dirac or Bose-Einstein distributions

$$f_{\rm EQ}(\mathbf{p}, \mu_X) = \left[\exp((E_X - \mu_X)/T) \pm 1\right], \tag{8}$$

where $E_X = \left(\mathbf{p}^2 + m_X^2\right)^{1/2}$ is the energy, μ_X is the chemical potential of the species and $+1$ pertains to the Fermi-Dirac species and -1 to the Bose-Einstein species.

In the relativistic regime $T \gg m_X, \mu_X$ formula (7) reduces to

$$n_X^{\rm EQ} = \left\{ \begin{array}{cc} (\zeta(3)/\pi^2) g_X T^3 & ({\rm Bose}), \\ (3/4)(\zeta(3)/\pi^2) g_X T^3 & ({\rm Fermi}), \end{array} \right. \tag{9}$$

where $\zeta(3) \simeq 1.2$ is the Riemann function of 3. In the non-relativitic limit, $T \ll m_X$, the number density is the same for Bose and Fermi species and reads

$$n_X^{\rm EQ} = g_X \left(\frac{m_X T}{2\pi}\right)^{3/2} e^{-\frac{m_X}{T} + \frac{\mu_X}{T}}. \tag{10}$$

It is also important to define the number density of particles minus the number density of antiparticles

$$\begin{aligned} n_X^{\rm EQ} - n_{\overline{X}}^{\rm EQ} &= \frac{g_X}{(2\pi)^3} \int f_{\rm EQ}(\mathbf{p}, \mu_X)\, d^3p - (\mu_X \leftrightarrow -\mu_X) \\ &= \left\{ \begin{array}{cc} \frac{g_X T^3}{6\pi^2} \left[\pi^2 \left(\frac{\mu_X}{T}\right) + \left(\frac{\mu_X}{T}\right)^3\right] & (T \gg m_X) \\ 2 g_X (m_X T/2\pi)^{3/2} \sinh(\mu_X/T) \exp(-m_X/T) & (T \ll m_X). \end{array} \right. \end{aligned} \tag{11}$$

Notice that, in the relativistic limit $T \gg m_X$, this difference scales linearly for $T \gtrsim \mu_X$. This means that detailed balances among particle number asymmetries may be expressed in terms of linear equations in the chemical potentials.

We can similarly define the equilibrium energy density $\rho_X^{\rm EQ}$ of a species X as

$$\rho_X^{\rm EQ} = \frac{g_X}{(2\pi)^3} \int E\, f_{\rm EQ}(\mathbf{p})\, d^3p, \tag{12}$$

which reads in the relativistic limit

$$\rho_X^{\rm EQ} = \left\{ \begin{array}{cc} (\pi^2/30) g_X T^4 & ({\rm Bose}), \\ (7/8)(\pi^2/30) g_X T^4 & ({\rm Fermi}). \end{array} \right. \tag{13}$$

Since the energy density of a non-relativistic particle species is exponentially smaller than that of a relativisitic species, it is a very convenient approximation to include only relativistic species with energy density ρ_R in the total energy density ρ of the Universe at temperature T

$$\rho \simeq \rho_R = \frac{\pi^2}{30} g_* T^4, \tag{14}$$

where g_* counts the total number of effectively massless degrees of freedom of the plasma

$$g_* = \sum_{i={\rm bos}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i={\rm fer}} g_i \left(\frac{T_i}{T}\right)^4. \tag{15}$$

Here T_i denotes the effective temperature of any species i (which might be decoupled from the thermal bath at temperature T). In the rest of these lectures we will be always concern with temperatures higher than about 100 GeV. At these temperatures, all the degrees of freedom of the standard model are in equilibrium and g_* is at least equal to 106.75.

From this expression we derive that, when the energy density of the Universe was dominated by a gas of relativistic particles, $\rho \propto a^{-4} \propto T^4$ and, therefore [73]

$$T \propto a^{-1}. \tag{16}$$

Assuming that during the early radiation-dominated epoch ($t \lesssim 4\times10^{10}$ sec), the scale factor scales like t^α, where α is a constant, the Hubble parameter scales like $t^{-1} \propto T^2 \propto a^{-2}$. This means that the scale factor $a(t)$ scales like $t^{1/2}$ and we recover *2)* of Eq. (6). More precisely, the expansion rate of the Universe H is [73]

$$H = \left(\frac{8\pi}{3M_{\rm P}^2}\rho\right)^{1/2} \simeq 1.66\, g_*^{1/2}\frac{T^2}{M_{\rm P}}. \tag{17}$$

Using the fact that $H = (1/2t)$ and Eq. (17), we can easily relate time and temperature as

$$t \simeq 0.301 \frac{M_{\rm P}}{g_*^{1/2} T^2} \simeq \left(\frac{T}{\rm MeV}\right)^{-2} \;\text{sec}. \tag{18}$$

Another quantity that will turn out to be useful in the following is the entropy density. Throughout most of the history of the Universe, local thermal equilibrium is attained and the entropy in a comoving volume element s remains constant. Since it is dominated by the contribution of relativisitic particles, to a very good approximation

$$s = \frac{2\pi^2}{45} g_{*S} T^3, \tag{19}$$

where

$$g_{*S} = \sum_{i={\rm bos}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i={\rm fer}} g_i \left(\frac{T_i}{T}\right)^3. \tag{20}$$

For most of the history of the Universe, however, all the particles have the same temperature and we can safely replace g_{*S} with g_*. Notice that the conservation of entropy implies that $s \propto a^{-3}$ and therefore $g_{*S}T^3a^3$ remains a constant as the Universe expands. This means that the number of some species X in a comoving volume $N_X \equiv a^3 n_X$ is proportional to the number density of that species divided by s, $N_X \propto n_X/s$.

2.2 Local thermal equilibrium and chemical equilibrium

So far we have been using the fact that, throughout most of the history of the Universe, thermal equilibrium was attained. The characteristic time τ_X for particles of a species X with respect to the process $X + A \cdots \to C + D + \cdots$ is defined by the rate of change of the number of particles per unit volume n_X due to this process

$$\frac{1}{\tau_X} = -\frac{1}{n_X}\left(\frac{dn_X}{dt}\right)_{X+A\cdots\to C+D+\cdots}. \tag{21}$$

In the early Universe, if τ_X is smaller than the characteristic time of the expansion H^{-1}, then there is enough time for the process to occur and the particles X's are said to be *thermally* coupled to the cosmic fluid. By contrast, if $\tau_X \gg H^{-1}$, for *every* process in which the particles X's are involved, then they are not in thermal equilibrium and they are said to be decoupled.

In order to analyze the evolution of the particle populations which constitute the cosmic fluid, it is necessary to compare H^{-1} with τ_X at different temperatures. This is done through the Boltzmann equation [8], which, in an expanding Universe, reads

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_X) = \int \pi_X\, C[f_X], \tag{22}$$

where C is the collision operator. Eq. (22) may be rewritten as

$$\begin{aligned} \frac{dn_X}{dt} \;&+\; 3Hn_X = \sum_{j,\ell,m,\cdots} \int \pi\, f_\ell f_m \cdots (1 \pm f_X)(1 \pm f_j) \cdots W\left(\ell + m + \cdots \to X + j + \cdots\right) \\ &-\; f_X f_j \cdots (1 \pm f_\ell)(1 \pm f_m) \cdots W\left(X + j + \cdots \to \ell + m + \cdots\right), \end{aligned} \tag{23}$$

where $\pi \equiv \pi_X \pi_j \cdots \pi_\ell \pi_m \cdots$, $\pi_i = (2\pi)^{-3} g_i (d^3p/2E_i)$ is the volume element in the phase space, W is the matrix element of the given process and $(+)$ applies to bosons and $(-)$ to fermions.. The second term in the left-hand side of Eq. (23) accounts for the n_X diluition due to the cosmic expansion and the right-hand side accounts for the n_X variations due to any elemenatry process $X + j + \cdots \to \ell + m + \cdots$ in which the X particles are involved. As it stands, Eq. (23) is rather formidable and complicated, but some approximations can be made to transform it in a simpler form.

Let us consider, for example, a process like $X + f \to X' + f$, where the number of X particles does change in the scatterings and let us also suppose that the f particles are light $(T \gg m_f)$ and that the corresponding population is in thermal equilibrium. In the case in which the X distribution function is described by a Maxwell-Boltzmann distribution, *i.e.* the X particles are in equilibrium at temperatures smaller than m_X, it is easy to see that the right-hand side of Eq. (23) may be expressed in the form

$$\text{r.h.s. of Eq. (23)} = -\left[n_X - n_X^{\rm EQ}\right] S \tag{24}$$

where

$$\begin{aligned} S \;&=\; \int \pi_f\, E_f\, f_f^{\rm EQ}\, \sigma(X + f \to X' + f) \\ &\simeq\; n_f^{\rm EQ}\, \langle \sigma(X + f \to X' + f)\, v \rangle. \end{aligned} \tag{25}$$

The notation $\langle \sigma v \rangle$ stands for the thermal average cross section times the relative velocity v. The inverse time scale $\tau_X^{-1} = \Gamma_X$ associated to the elastic process is therefore

$$\Gamma_X = \tau_X^{-1} \simeq n_f^{\rm EQ} \langle \sigma(X + f \to X' + f) v \rangle. \tag{26}$$

From these very simple considerations, we may conclude that the X degrees of freedom are in thermal equilibrium if

$$\Gamma_X \simeq n_f^{\rm EQ} \langle \sigma(X + f \to X' + f) v \rangle \gtrsim H \qquad \text{(thermal equilibrium is attained)}. \tag{27}$$

Departure from thermal equilibrium is expected whenever a rate crucial for mantaining thermal equilibrium becomes smaller than the expansion rate, $\Gamma_X \lesssim H$.

Another useful concept is that of *chemical* equilibrium. In general, a species X is in chemical equilibrium if the inelastic scatterings which change the number of X particles in the plasma, $X + j \to \ell + m$, have a rate $\Gamma_{\rm inel}$ larger than the expansion rate of the Universe. In such a case, one is allowed to write down a relation between the different chemical potentials μ's

$$\mu_X + \mu_j = \mu_\ell + \mu_m \tag{28}$$

of the particles involved in the process. With these simple notions in mind we may start our voyage towards the country of baryogenesis.

3 The graveyard for a baryon symmetric Universe

The CPT theorem assures that any particle species X there exists the antiparticle $\overline{X}$ with exactly the same mass, $m_X = m_{\overline{X}}$, and decay width, $\Gamma_X = \Gamma_{\overline{X}}$, and eventually opposite charges associated to these particles, $Q_X = -Q_{\overline{X}}$. This striking symmetry would naturally lead us to conclude that the Universe contains particles and antiparticles in equal number densities, $n_X = n_{\overline{X}}$. The observed Universe, however, is drastically different. We do not observe any bodies of antimatter within the solar system and only antiprotons $\overline{p}$ in the cosmic rays, which are believed to be of extra solar origin. Antiprotons are likely to be produced as secondaries in collisions $pp \to 3p + \overline{p}$ at a rate similar to the observed one

$$\frac{n_{\bar{p}}}{n_p} \sim 3 \times 10^{-4}. \tag{29}$$

The experimental limit on $\overline{n}_{^4{\rm He}}/n_{^4{\rm He}}$ is similarly of the order of 10^{-5}. We cannot exclude, of course, that the dominance of matter over antimatter is only local and is only realized up to a certain length scale ℓ_B, beyond which the picture is reversed and islands of antimatter are found. However, the size of our matter domain must be quite large, roughly speaking $\ell_B \gtrsim$ 10 Mpc [118, 119] (for more restrictive bounds see [36]). Indeed, for smaller scales one would expect a significant amount of energetic γ-rays coming from the reaction of annihilation of $p\overline{p}$ into π-mesons followed by the subsequent decay $\pi^0 \to 2\gamma$, which would take in the boundary area separating the matter and antimatter islands. Another signature for the presence of domains of antimatter would be the distortion of the spectrum of the cosmic microwave background radiation. In such a case, the permitted value of ℓ_B might be smaller if voids separate matter and antimatter domains. These voids might be created because of an excessive pressure produced by the annihilations at earlies stages of the evolution of the Universe or because of low density matter and antimatter in the boundary regions, provided that the baryon asymmetry changes sign locally so that in the boundaries it is zero or very small.

All these considerations lead us to conclude that, if domains of matter and antimatter exist in the Universe, they are separated on scales certainly larger than the radius of our own galaxy ($\sim$ 3 Kpc) and most probably on scales larger than the Virgo cluster ($\sim$ 10 Mpc). A much more severe bound on ℓ_B ($\sim$ 300 Mpc) is potentially reachable by the Alpha Magnetic Spectrometer (AMS) [7], a detector for extraterrestrial study of antimatter, matter and missing matter which, after a precursor flight on STS91 in May 1998, will be installed on the International Space Station where it should operate for three years.

3.1 Some considerations on nucleosynthesis and the baryon number

The baryon number density does not keep constant during the evolution of the Universe because it scales like a^{-3}, where a is the cosmological scale factor [73]. It is therefore convinient to define the baryon asymmetry of the Universe in terms of the quantity

$$\eta \equiv \frac{n_B}{n_\gamma}, \tag{30}$$

where $n_B = n_b - n_{\bar{b}}$ is the difference between the number of baryons and antibaryons per unit volume and $n_\gamma = 2\frac{\zeta(3)}{\pi^2}T^3$ is the photon number density at a certain temperature T. The parameter η is essential for determining the present light element abundances produced at the nucleosynthesis epoch. The parameter η may have not changed since nucleosynthesis. At these energy scales (~ 1 MeV) the baryon number is conserved if there are no processes which would have produced entropy to change the photon number.

Let us now estimate η. The baryon number density is

$$n_B = \frac{\rho_B}{m_B} = \frac{\Omega_B}{m_B}\,\rho_c, \tag{31}$$

where ρ_B is the baryonic energy density and $\Omega_B \equiv \rho_B/\rho_c$. Using the critical density

$$\rho_c = 1.88 \times 10^{-29}\; h^2 \;\mathrm{gr\;cm}^{-3}, \tag{32}$$

where $0.5 \lesssim h \lesssim 0.9$ parametrizes the present value of the Hubble parameter H_0, $h \equiv H/100$ Km Mpc^{-1} sec $^{-1}$, we obtain

$$n_B = 1.1 \times 10^{-5}\; h^2\; \Omega_B \;\mathrm{cm}^{-3}. \tag{33}$$

On the other hand, the present temperature of the background radiation is $T_0 = 2.735\;{}^0$K giving rise to

$$n_\gamma \simeq 415\; \left(\frac{T_0}{2.735\;{}^0K}\right)^3\; \mathrm{cm}^{-3}. \tag{34}$$

Putting (33) and (34) together, we obtain

$$\eta = 2.65 \times 10^{-8}\;\Omega_B\; h^2\; \left(\frac{T_0}{2.735\;{}^0K}\right)^{-3}. \tag{35}$$

The range of η consistent with the deuterium and ^{3}He primordial abundances is [73]

$$4(3) \times 10^{-10} \lesssim \eta \lesssim 7(10) \times 10^{-10}, \tag{36}$$

where the most conservative bounds are in parenthesis. Conversely we may write the range for $\Omega_B\, h^2$ to be

$$0.015(0.011) \lesssim \Omega_B\; h^2 \lesssim 0.026(0.038). \tag{37}$$

Sometimes it is useful to describe the baryon asymmetry in terms of $B \equiv n_B/s$, where s is the entropy density of the Universe at a certain temperature T. The range (36) translates into

$$5.7(4.3) \times 10^{-11} \lesssim B \lesssim 9.9(14) \times 10^{-11}. \tag{38}$$

Now, the fundamental question is: are we able to explain the tiny value of η within the standard cosmological model?

Suppose that initially we start with $\eta = 0$. We can compute the final number density of nucleons b that are left over after annihilations have frozen out. At temperatures $T \lesssim 1$ GeV the equilibrium abundance of nucleons and antinucleons is [73]

$$\frac{n_b}{n_\gamma} \simeq \frac{n_{\bar{b}}}{n_\gamma} \simeq \left(\frac{m_p}{T}\right)^{3/2} e^{-\frac{m_p}{T}}. \tag{39}$$

When the Universe cools off, the number of nucleons and antinucleons decreases as long as the annihilation rate $\Gamma_{\rm ann} \simeq n_b \langle \sigma_A v \rangle$ is larger than the expansion rate of the Universe $H \simeq 1.66\, g_*^{1/2} \frac{T^2}{M_{\rm P}}$. The thermally averaged annihilation cross section $\langle \sigma_A v \rangle$ is of the order of m_π^2. At $T \simeq 20$ MeV, $\Gamma_{\rm ann} \simeq H$ and annihilations freeze out, nucleons and antinucleons being so rare that they cannot annihilate any longer. Therefore, from (39) we obtain

$$\frac{n_b}{n_\gamma} \simeq \frac{n_{\bar{b}}}{n_\gamma} \simeq 10^{-18}, \tag{40}$$

which is much smaller than the value required by nucleosynthesis. In order to avoid the annihilation catastrophe, we may suppose that hypothetical new interactions separated matter from antimatter before $T \simeq 38$ MeV, when $\eta \simeq 10^{-10}$. At that time, $t \simeq 10^{-3}$ sec, however, the causal region (horizon) was small and contained only $\sim 10^{-7} M_\odot$. Hence we cannot explain the asymmetry over the galaxy scales. This argument is not valid, however, in cosmological models invoking inflation. Indeed, in these models the region of the Universe which is causally connected today was connected even at times $\sim 10^{-3}$ sec. These scenarios pose other serious cosmological drawbacks, though. If the processes responsible for the separation of matter from antimatter took place before inflation, then the baryon number was diluted by an enormous factor $\sim \exp(200)$, because of the entropy production due to inflation. On the other side, if the separation took place after inflation, then it is not clear how to eliminate the boundaries separating matter from antimatter islands.

Another possibility may be represented by explaining the tiny value of η via statistical fluctuations in the baryon and antibaryon distributions. Our own galaxy containes at the present epoch approximately 10^{79} photons. The comoving volume V that encompasses our galaxy today contains about 10^{69} baryons, but when the temperature was $T \gtrsim 1$ GeV, it contained about 10^{79} baryons and antibaryons. Frome pure statistical fluctuations one may expect an asymmetry $(n_b - n_{\bar{b}})/n_b \simeq (n_b V)^{-1/2} \simeq 10^{-39.5}$, which is again far too small to explain the observed baryon asymmetry.

In conclusion, in the standard cosmological model there is no explanation for the smallness of the ratio (36), if we start from $\eta = 0$. An initial asymmetry may be imposed by hand as an initial condition, but this would violate any naturalness principle and would be extremely boring!

3.2 The three basic conditions for baryogenesis

As we have already learned, the Universe was initially baryon symmetric ($n_b = n_{\bar{b}}$) although the matter-antimatter asymmetry appears to be large today ($n_b \gg n_{\bar{b}}$). In the standard cosmological model there is no explanation for such a small value of the baryon asymmetry consistent with nucleosynthesis and it has to be imposed by hand as an initial condition. This option is far from being appealing. However, it has been suggested by Sakharov long ago [114] that a tiny baryon asymmetry B may have been produced in the early Universe. Three are the necessary conditions for this to happen.

Exercise 1

Show that the baryon asymmetry is zero if there is no baryon number violation.

3.2.1 Baryon number violation

This condition is somehow obvious since we want to start from a baryon symmetric Universe ($B = 0$) and to evolve it to a Universe where $B \neq 0$. Baryon number violation interactions are therefore mandatory. They might also mediate proton decay; in such a case phenomenological constraints are provided by the lower bound on the proton lifetime $\tau_p \gtrsim 5 \times 10^{32}$ years..

3.2.2 C and CP violation

C (charge conjugation symmetry) and CP (the product of charge conjugation and parity) are not exact symmetries. Indeed, were C an exact symmetry, the probability of the process $i \rightarrow f$ would be equal to the one of the process $\bar{i} \rightarrow \overline{f}$. Since the baryon number of f is equal in absolute value and opposite in sign to that of $\overline{f}$, the net baryon number B would vanish. C is maximally violated by the weak interactions.

Furthermore, because of the CPT theorem, CP invariance is equivalent to time-invariance (time reversal). The latter assures that the rate of the process

$$i(\mathbf{r}_i, \mathbf{p}_i, \mathbf{s}_i) \rightarrow f(\mathbf{r}_j, \mathbf{p}_j, \mathbf{s}_j) \tag{41}$$

and that of its time-reversed process

$$f(\mathbf{r}_j, -\mathbf{p}_j, -\mathbf{s}_j) \rightarrow i(\mathbf{r}_i, -\mathbf{p}_i, -\mathbf{s}_i) \tag{42}$$

are equal. Thus, even though it is possible to create a baryon asymmetry in a certain region of the phase space, integrating over all momenta $\mathbf{p}$ and summing over all spins $\mathbf{s}$ would produce a vanishing baryon asymmetry. CP violation has been observed in the kaon system. However, a fundamental understanding of CP violation is still lacking. Hopefully, studies of baryogenesis may shed some light on it.

3.2.3 Departure from thermal equilibrium

If all the particles in the Universe remained in thermal equilibrium, then no preferred direction for time may be defined and the CPT invariance would prevent the appearance of any baryon excess, making the presence of CP violating interactions irrelevant.

Let us suppose that a certain species X with mass m_X is in thermal equilibrium at temperatures $T \ll m_X$. Its number density will be given by

$$n_X \simeq g_X (m_X T)^{3/2}\, e^{-\frac{m_X}{T}+\frac{\mu_X}{T}}, \tag{43}$$

where μ_X is the associated chemical potential.

As we have mentioned in the previous Section, a species X is in *chemical* equilibrium if the inelastic scatterings which change the number of X particles in the plasma, $X + A \rightarrow B + C$, have a rate $\Gamma_{\rm inel}$ larger than the expansion rate of the Universe. In such a case, one can write down a relation among the different chemical potentials of the particles involved in the process

$$\mu_X + \mu_A = \mu_B + \mu_C. \tag{44}$$

In this way the number density in thermal equilibrium of the antiparticle $\overline{X}$ ($m_X = m_{\bar{X}}$) is

$$n_{\bar{X}} \simeq g_X (m_X T)^{3/2}\, e^{-\frac{m_X}{T}-\frac{\mu_X}{T}}, \tag{45}$$

where we have made use of the fact that $\mu_{\bar{X}} = -\mu_X$ because of the process

$$\overline{X}X \rightarrow \gamma\gamma, \tag{46}$$

and $\mu_\gamma = 0$. If the X particle carries baryon number, then B will get a contribution from

$$B \propto n_X - n_{\bar{X}} = 2 g_X (m_X T)^{3/2} e^{-\frac{m_X}{T}} \sinh\left(\frac{\mu_X}{T}\right). \tag{47}$$

The crucial point is now that, if X and $\overline{X}$ undergo B-violating reactions, as required by the first Sakharov condition,

$$XX \rightarrow \overline{XX}, \tag{48}$$

then $\mu_X = 0$ and the relative contribution of the X particles to the net baryon number vanishes. Only a departure from thermal equilibrium can allow for a finite baryon excess.

4 The standard out-of-equilibrium decay scenario

Out of the three Sakharov conditions that we discussed in the previous section, the baryon number violation and C and CP violation may be investigated thoroughlly only within a given particle physics model, while the third condition – the departure from thermal equilibrium –may be discussed in a more general way. Very roughly speaking, the various models of baryogenesis that have been proposed so far fall into two categories:

– models where the out-of-equilibrium condition is attained thanks to the expansion of the Universe and the presence of heavy decaying particles;

– models where the departure from thermal equilibrium is attained during the phase transitions which lead to the breaking of some global and/or gauge symmetry.

In this lecture we will analyse the first category –the standard out-of-equilibrium decay scenario [73, 72].

4.1 The conditions for the out-of-equilibrium decay scenario

It is obvious that in a static Universe any particle, even very weakly interacting, will attain sooner or later thermodynamical equilibrium with the surroinding plasma. The expansion of the Universe, however, introduces a finite time-scale, $\tau_U \sim H^{-1}$. Let suppose that X is a *baryon number violating* superheavy boson field (vector or scalar) which is coupled to lighter fermionic degrees of freedom with a strength $\alpha_X^{1/2}$ (either a gauge coupling $\alpha_{\rm gauge}$ or a Yukawa coupling α_Y).

In the case in which the couplings are renormalizable, the decay rate Γ_X of the superheavy boson may be easily estimated to be

$$\Gamma_X \sim \alpha_X\, M_X, \tag{49}$$

where M_X is the mass of the particle X. In the opposite case in which the boson is a gauge *singlet* scalar field and it only couples to light matter through gravitational interactions – this is the case of singlets in the hidden sector of supergravity models [103] – the decay rate is from dimensional arguments

$$\Gamma_X \sim \frac{M_X^3}{M_{\rm P}^2}. \tag{50}$$

At very large temperatures $T \gg M_X$, it is assumed that all the particles species are in thermal equilibrium, *i.e.* $n_X \simeq n_{\overline{X}} \simeq n_\gamma$ (up to statistical factors) and that $B = 0$. At $T \lesssim M_X$ the equilibrium abundance of X and $\overline{X}$ relative to photons is given by

$$\frac{n_X^{\rm EQ}}{n_\gamma} \simeq \frac{n_{\overline{X}}^{\rm EQ}}{n_\gamma} \simeq \left(\frac{M_X}{T}\right)^{3/2} e^{-\frac{M_X}{T}}, \tag{51}$$

where we have neglected the chemical potential μ_X.

For the X and $\overline{X}$ particles to mantain their equilibrium abundances, they must be able to diminish their number rapidly with respect to the Hubble rate $H(T)$. The conditions necessary for doing so are easily quantified. The superheavy X and $\overline{X}$ particles may attain equilibrium through decays with rate Γ_X, inverse decays with rate $\Gamma_X^{\rm ID}$

$$\Gamma_X^{\rm ID} \simeq \Gamma_X \left\{ \begin{array}{cc} 1 & T \gtrsim M_X, \\ (M_X/T)^{3/2}\exp(-M_X/T) & T \lesssim M_X, \end{array} \right. \tag{52}$$

and annihilation processes with rate $\Gamma_X^{\rm ann} \propto n_X$. The latter, however are "self-quenching" and therefore less important than the decay and inverse decay processes. They will be ignored from now on. Of crucial interest are the B-nonconserving scattering processes $2 \leftrightarrow 2$ mediated by the X and $\overline{X}$ particles with rate $\Gamma_X^{\rm S}$

$$\Gamma_X^{\rm S} \simeq n\sigma \simeq \alpha^2 T^3 \frac{T^2}{(M_X^2 + T^2)^2}, \tag{53}$$

where $\alpha \simeq g^2/4\pi$ denotes the coupling strength of the X boson. At high temperatures, the $2 \leftrightarrow 2$ scatterings cross section is $\sigma \simeq \alpha^2/T^2$, while at low temperatures $\sigma \simeq \alpha^2 T^2/M_X^4$.

For baryogenesis, the most important rate is the decay rate, as decays (and inverse decays) are the mechanism that regulates the number of X and $\overline{X}$ particles in the plasma. It is therefore useful to define the following quantity

$$K \equiv \left.\frac{\Gamma_X}{H}\right|_{T=M_X} \tag{54}$$

which measures the effectiveness of decays at the crucial epoch ($T \sim M_X$) when the X and $\overline{X}$ particles must decrease in number if they are to stay in equilibrium. Note also that for $T \lesssim M_X$, K determines the effectiveness of inverse decays and $2 \leftrightarrow 2$ scatterings as well: $\Gamma_X^{\rm ID}/H \simeq (M_X/T)^{3/2}\exp(-M_X/T)\,K$ and $\Gamma_X^{\rm S}/H \simeq \alpha(T/M_X)^5\,K$.

Now, if $K \gg 1$, and therefore

$$\Gamma_X \gg H|_{T=M_X}\,, \tag{55}$$

then the X and $\overline{X}$ particles will adjust their abundances by decaying to their equilibrium abundances and no baryogenesis can be induced by their decays –this is simply because out-of-equilibrium conditions are not attained. Given the expression (17) for the expansion rate of the Universe, the condition (55) is equivalent to

$$M_X \ll g_*^{-1/2}\,\alpha_X\,M_{\rm P} \tag{56}$$

for strongly coupled scalar bosons, and to

$$M_X \gg g_*^{1/2}\,M_{\rm P}, \tag{57}$$

for gravitationally coupled X particles. Obviously, this last condition is never satisfied for $M_X \lesssim M_{\rm P}$.

However, if the decay rate is such that $K \ll 1$, and therefore

$$\Gamma_X \lesssim H|_{T=M_X}\,, \tag{58}$$

then the X and $\overline{X}$ particles cannot decay on the expansion time-scale τ_U and so *they remain as abundant as photons* for $T \lesssim M_X$. In other words, at some temperature $T > M_X$, the superheavy bosons X and $\overline{X}$ are so weakly interacting that they cannot catch up with the expansion of the Universe and they decouple from the thermal bath when still *relativistic*, $n_X \simeq n_{\overline{X}} \simeq n_\gamma$ at the time of decoupling. Therefore, at temperature $T \simeq M_X$, they will populate the Universe with an abundance which is much larger than the equilibrium one. This *overbundance* with respect to the equilibrium abundance is precisely the departure from thermal equilibrium needed to produce a final nonvanishing baryon asymmetry. Condition (58) is equivalent to

$$M_X \gtrsim g_*^{-1/2}\,\alpha_X\,M_{\rm P} \tag{59}$$

for strongly coupled scalar bosons, and to

$$M_X \lesssim g_*^{1/2}\,M_{\rm P}, \tag{60}$$

for gravitationally coupled X particles. It is clear that this last condition is always satisfied, whereas the condition (59) is based on the smallness of the quantity $g_*^{-1/2}\alpha_X$. In particular, if the X particle is a gauge boson, $\alpha_X \sim \alpha_{\rm gauge}$ can span the range $(2.5\times 10^{-2}-10^{-1})$, while g_* is about 10^2. In this way we obtain from (59) that the condition of out-of-equilibrium can be satisfied for

$$M_X \gtrsim (10^{-4}-10^{-3})\, M_{\rm P} \simeq (10^{15}-10^{16})\ {\rm GeV}. \tag{61}$$

If X is a scalar boson, its coupling α_Y to fermions f with mass m_f is proportional to the squared mass of the fermions

$$\alpha_Y \sim \left(\frac{m_f}{m_W}\right)^2 \alpha_{\rm gauge}, \tag{62}$$

where m_W is the W-boson mass and α_Y is typically in the range $(10^{-2}-10^{-7})$, from where

$$M_X \gtrsim (10^{-8}-10^{-3})\, M_{\rm P} \simeq (10^{10}-10^{16})\ {\rm GeV}. \tag{63}$$

Obviously, condition (63) is more easily satisfied than condition (61) and we conclude that baryogenesis is more easily produced through the decay of superheavy scalar bosons. On the other hand, as we have seen above, the condition (60) tells us that the out-of-equilibrium condition is automatically satisfied for gravitationally interacting particles.

4.2 The production of the baryon asymmetry

Let us now follow the subsequent evolution of the X and $\overline{X}$ particles. When the Universe becomes as old as the lifetime of these particles, $t \sim H^{-1} \sim \Gamma_X^{-1}$, they start decaying. This takes place at a temperature T_D defined by the condition

$$\Gamma_X \simeq H|_{T=T_D}\,, \tag{64}$$

i.e. at

$$T_D \simeq g_*^{-1/4}\, \alpha_X^{1/2}\, (M_X M_{\rm P})^{1/2} < M_X, \tag{65}$$

where the last inequality comes from (59) and is valid for particles with unsuppressed couplings. For particles with only gravitational interactions

$$T_D \sim g_*^{-1/4}\, M_X \left(\frac{M_X}{M_{\rm P}}\right)^{1/2} < M_X, \tag{66}$$

the last inequality coming from (60). At $T \sim T_D$, X and $\overline{X}$ particles start to decay and their number decrease. If their decay violate the baryon number, they will generate a net baryon number per decay.

Suppose now that the X particle may decay into two channels, let us denote them by a and b, with different baryon numbers B_a and B_b, respectively. Correspondingly, the decay channels of $\overline{X}$, $\overline{a}$ and $\overline{b}$, have baryon numbers $-B_a$ and $-B_b$, respectively. Let $r(\overline{r})$ be the branching ratio of the $X(\overline{X})$ in channel $a(\overline{a})$ and $1-r(\overline{r})$ the branching ratio of $X(\overline{X})$ in channel $b(\overline{b})$,

$$r \;=\; \frac{\Gamma(X\to a)}{\Gamma_X},$$

$$
\begin{aligned}
\overline{r} &= \frac{\Gamma(\overline{X} \to \overline{a})}{\Gamma_X}, \\
1 - r &= \frac{\Gamma(X \to b)}{\Gamma_X}, \\
1 - \overline{r} &= \frac{\Gamma(\overline{X} \to \overline{b})}{\Gamma_X},
\end{aligned} \tag{67}
$$

where we have been using the fact that the *total* decay rates of X and $\overline{X}$ are equal because of the CPT theorem plus unitarity.

The average net baryon number produced in the X decays is

$$
rB_a + (1-r)B_b, \tag{68}
$$

and that produced by $\overline{X}$ decays is

$$
-\overline{r}B_a - (1-\overline{r})B_b. \tag{69}
$$

Finally, the mean net baryon number produced in X and $\overline{X}$ decays is

$$
\Delta B = (r-\overline{r})B_a + \left[(1-r)-(1-\overline{r})\right]B_b = (r-\overline{r})(B_a - B_b). \tag{70}
$$

Equation (70) may be easily generalized to the case in which $X(\overline{X})$ may decay into a set of final states $f_n(\overline{f}_n)$ with baryon number $B_n(-B_n)$

$$
\Delta B = \frac{1}{\Gamma_X}\sum_n B_n \left[\Gamma(X \to f_n) - \Gamma(\overline{X} \to \overline{f}_n)\right]. \tag{71}
$$

At the decay temperature, $T_D \lesssim M_X$, because $K \ll 1$ both inverse decays and $2 \leftrightarrow 2$ baryon violating scatterings are impotent and can be safely ignored and thus the net baryon number produced per decay ΔB is not destroyed by the net baryon number $-\Delta B$ produced by the inverse decays and by the baryon number violating scatterings.

At $T \simeq T_D$, $n_X \simeq n_{\overline{X}} \simeq n_\gamma$ and therefore the net baryon number density produced by the out-of-equilibrium decay is

$$
n_B = \Delta B \, n_X, \tag{72}
$$

from where we can see that ΔB coincides with the parameter η defined in (30) if $n_X \simeq n_\gamma$.

The three Sakharov ingredients for producing a net baryon asymmetry can be easily traced back here:

– If B is not violated, then $B_n = 0$ and $\Delta B = 0$.

–If C and CP are not violated, then $\Gamma(X \to f_n) = \Gamma(\overline{X} \to \overline{f}_n)$, and also $\Delta B = 0$.

– In thermal equilibrium, the inverse processes are not suppressed and the net baryon number produced by decays will be erased by the inverse decays.

Since each decay produces a mean net baryon number density $n_B = \Delta B n_X \simeq \Delta B n_\gamma$ and since the entropy density is $s \simeq g_* n_\gamma$, the net baryon number produced is

$$
B \equiv \frac{n_B}{s} \simeq \frac{\Delta B n_\gamma}{g_* n_\gamma} \simeq \frac{\Delta B}{g_*}. \tag{73}
$$

Taking $g_* \sim 10^2$, we see that only tiny C and CP violations are required to generate $\Delta B \sim 10^{-8}$, and thus $B \sim 10^{-10}$.

To obtain (73) we have assumed that the entropy realese in X decays is negligible. However, sometimes, this is not a good approximation (especially if the X particles decay very late, at $T_D \ll M_X$, which is the case of gravitationally interacting particles). In that case, assuming that the energy density of the Universe at T_D is dominated by X particles

$$\rho_X \simeq M_X\, n_X, \tag{74}$$

and that it is converted entirely into radiation at the reheating temperature T_{RH}

$$\rho = \frac{\pi^2}{30}\, g_*\, T_{RH}^4, \tag{75}$$

we obtain

$$n_X \simeq \frac{\pi^2}{30}\, g_*\, \frac{T_{RH}^4}{M_X}. \tag{76}$$

We can therefore write the baryon number as

$$B \simeq \frac{3}{4}\frac{T_{RH}}{M_X}\Delta B. \tag{77}$$

We can relate T_{RH} with the decay rate Γ_X using the decay condition

$$\Gamma_X^2 \simeq H^2(T_D) \simeq \frac{8\pi\rho_X}{3M_{\rm P}^2} \tag{78}$$

and so we can write

$$B \simeq \left(\frac{g_*^{-1/2}\Gamma_X M_{\rm P}}{M_X^2}\right)^{1/2} \Delta B. \tag{79}$$

For the case of strongly decaying particles (through renormalizable interactions) we obtain

$$B \simeq \left(\frac{g_*^{-1/2}\alpha M_{\rm P}}{M_X}\right)^{1/2} \Delta B. \tag{80}$$

while for the case of weakly decaying particles (through gravitational interactions) we obtain

$$B \simeq \left(\frac{g_*^{-1/2} M_X}{M_{\rm P}}\right)^{1/2} \Delta B. \tag{81}$$

In the other extreme regime $K \gg 1$, one expects the abundance of X and $\overline{X}$ bosons to track the equilibrium values as $\Gamma_X \gg H$ for $T \sim M_X$. If the equilibrium is tracked precisely enough, there will be no departure from thermal equilibrium and no baryon number may evolve. The intermediate regime, $K \sim 1$, is more interesting and to address it one has to invoke numerical analysis involving Boltzmann equations for the evolution of B. This has been done in refs. [71, 46, 57, 72]. The numerical analysis essentially confirms the qualitative picture we have described so far and its discussion is beyond the scope of these lectures.

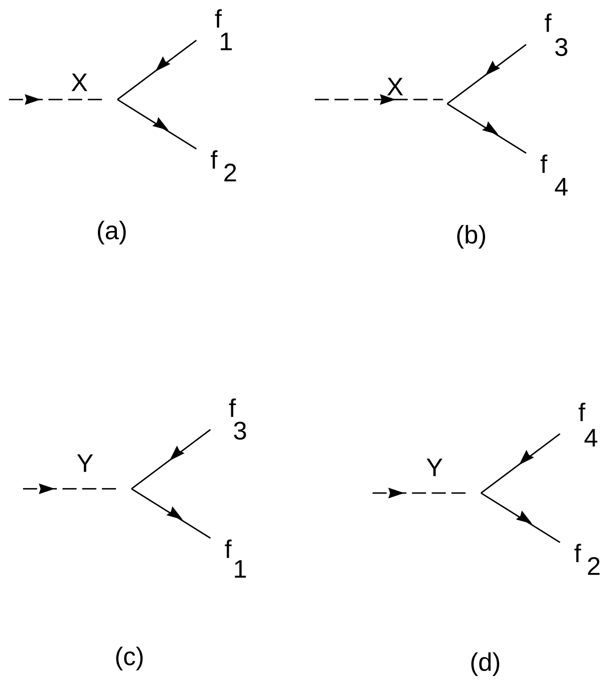


Figure 1: Couplings of X and Y to fermions f_i.

4.2.1 An explicit example

Let us consider first two massive boson fields X and Y coupled to four fermions f_1, f_2, f_3 and f_4 through the vertices of Fig. 1 and describing the decays $X \to \overline{f}_1 f_2, \overline{f}_3 f_4$ and $Y \to \overline{f}_3 f_1, \overline{f}_4 f_2$. We will refer to these vertices as $\langle f_2|X|f_1\rangle$, $\langle f_4|X|f_3\rangle$, $\langle f_1|Y|f_3\rangle$ and $\langle f_2|Y|f_4\rangle$, and their CP conjugate $\overline{X} \to \overline{f}_2 f_1, \overline{f}_4 f_3$ and $\overline{Y} \to \overline{f}_1 f_3, \overline{f}_2 f_4$ by their complex conjugate. In the Born approximation $\Delta B = 0$ because from (71) one finds

$$\Gamma(X \to \overline{f}_1 f_2)_{\rm Born} = I_X^{12} \left|\langle f_2|X|f_1\rangle\right|^2 = \Gamma(\overline{X} \to \overline{f}_2 f_1)_{\rm Born}, \tag{82}$$

where I_X^{12} accounts for the kinematic structures of the processes $X \to \overline{f}_1 f_2$ and $\overline{X} \to \overline{f}_2 f_1$ and the same may be found for the other processes contributing to ΔB. This shows that, to obtain a non-zero result for ΔB, one must include (at least) corrections arising from the interference of Born amplitudes of Fig. 1 with the one-loop amplitude of Fig. 2. For example, the interference of the diagrams in Fig. 1(a) and Fig. 2(a) (in the square amplitude) is shown in Fig. 3(a), where the thick dashed line is the unitarity cut (equivalent to say that each cut line represents on-shell mass particles). The amplitude of the diagram in Fig. 3(a) is given by $I_{XY}^{1234}\Omega_{1234}$, where the kinematic factor I_{XY}^{1234} accounts for the integration over the final state phase space of f_2 and $\overline{f}_1$ and over momenta of the internal states f_4 and $\overline{f}_3$, and

$$\Omega_{1234} = \langle f_1|Y|f_3\rangle^* \langle f_4|X|f_3\rangle \langle f_2|Y|f_4\rangle \langle f_2|X|f_1\rangle^*. \tag{83}$$

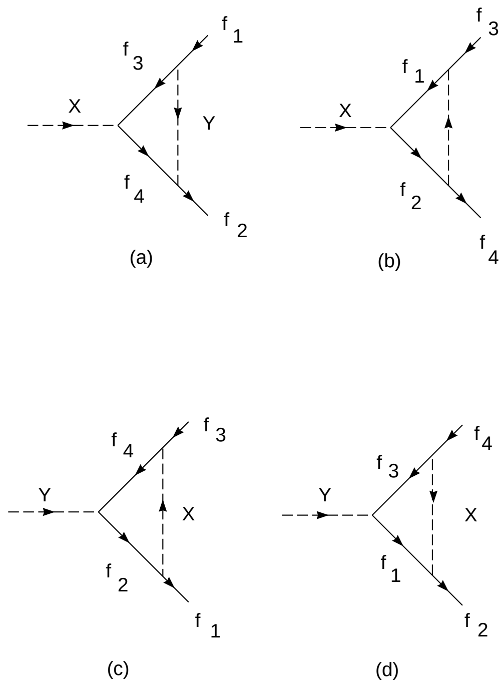


Figure 2: One-loop corrections to the Born amplitude of Fig. 1.

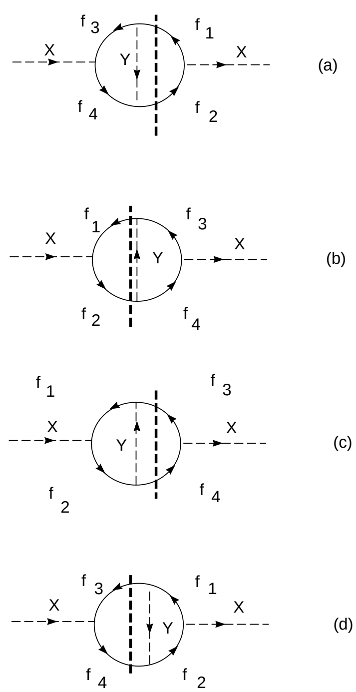


Figure 3: Intereference between the diagrams of Fig. 1 and Fig. 2 for the square amplitudes of X decay.

The complex conjugate diagram of Fig. 3(b) has the complex conjugate amplitude. Therefore, the contribution from the diagrams in Figs. 3(a) and 3(b) to the decay $X \to \overline{f}_1 f_2$ is

$$\Gamma(X \to \overline{f}_1 f_2)_{\rm interference} = I_{XY}^{1234} \Omega_{1234} + {\rm h.c.} \tag{84}$$

To obtain the CP conjugate amplitude $\overline{X} \to \overline{f}_2 f_1$ all couplings must be complex conjugated, although the kinematic factors I_{XY} are unaffected by CP conjugation. Therefore the interference contribution to the $\overline{X} \to \overline{f}_2 f_1$ decay rate is given by

$$\Gamma(\overline{X} \to \overline{f}_2 f_1)_{\rm interference} = I_{XY}^{1234} \Omega_{1234}^* + {\rm h.c.} \tag{85}$$

and the relevant quantity for baryogenesis is given by

$$\Gamma(X \to \overline{f}_1 f_2) - \Gamma(\overline{X} \to \overline{f}_2 f_1) = -4\,{\rm Im}\,\left[I_{XY}^{1234}\right]\,{\rm Im}\,\left[\Omega_{1234}\right]. \tag{86}$$

The diagrams of the decays $X \to \overline{f}_3 f_4$ and $\overline{X} \to \overline{f}_3 f_4$ differ from the one in Figs. 3(a) and 3(b) only in that the unitarity cut is taken through f_3 and f_4 instead of f_1 and f_2. One easily obtains

$$\Gamma(X \to \overline{f}_3 f_4) - \Gamma(\overline{X} \to \overline{f}_4 f_3) = -4\,{\rm Im}\,\left[I_{XY}^{3412}\right]\,{\rm Im}\,\left[\Omega_{1234}^*\right]. \tag{87}$$

The kinematic factors I_{XY} for loop diagrams may have an imaginary part whenever any internal lines may propagate on their mass shells in the intermediate states, picking the pole of the propagator

$$\frac{1}{p^2 - m^2 + i\epsilon} = \frac{\rm PP}{p^2 - m^2} + i\pi\delta(p^2 - m^2), \tag{88}$$

where PP stands for the principal part. This happens if $M_X > m_1 + m_2$ and $M_X > m_3 + m_4$. This means that with light fermions, the imaginary part of I_{XY} will be always nonzero. The kinematic factors ${\rm Im}\,[I_{XY}^{1234}]$ and ${\rm Im}\,[I_{XY}^{3412}]$ are therefore obtained from diagrams involving two unitarity cuts: one through the lines f_1 and f_2 and the other through the lines f_3 and f_4. The resulting quantities are invariant under the interchanges $f_1 \leftrightarrow f_3$ and $f_2 \leftrightarrow f_4$ and consequently

$${\rm Im}\,\left[I_{XY}^{1234}\right] = {\rm Im}\,\left[I_{XY}^{3412}\right] = {\rm Im}\,\left[I_{XY}\right]. \tag{89}$$

Defining B_i the baryon number of the fermion f_i, the net baryon number produced in the X decays is therefore

$$(\Delta B)_X = \frac{4}{\Gamma_X}\,{\rm Im}\,\left[I_{XY}\right]\,{\rm Im}\,\left[\Omega_{1234}\right]\left[B_4 - B_3 - (B_2 - B_1)\right]. \tag{90}$$

To compute the baryon asymmetry $(\Delta B)_Y$ one may observe that the set of vertices in Fig. 1 is invariant under the transformations $X \leftrightarrow Y$ and $f_1 \leftrightarrow f_4$. These rules yield

$$(\Delta B)_Y = \frac{4}{\Gamma_Y}\,{\rm Im}\,\left[I_{YX}\right]\,{\rm Im}\,\left[\Omega_{1234}^*\right]\left[B_4 - B_3 - (B_2 - B_1)\right] \tag{91}$$

and the total baryon number is therefore given by

$$(\Delta B) = (\Delta B)_X + (\Delta B)_Y = 4\left\{\frac{\text{Im }[I_{XY}]}{\Gamma_X} - \frac{\text{Im }[I_{YX}]}{\Gamma_Y}\right\} \text{Im }[\Omega_{1234}]\,[B_4 - B_3 - (B_2 - B_1)]\,. \tag{92}$$

We can notice a few things:

– If the X and Y couplings were B conserving, the two possible final states in X and Y decays would have the same baryon number, *i.e.* $B_4 - B_3 = B_2 - B_1$ and therefore $\Delta B = 0$. Therefore the baryon number must be violated not only in X decays but also in the decays of the particle exchanged in the loop.

– Some coupling constants in the Lagrangian must be complex to have Im $[\Omega_{1234}]$.

– Even if $(\Delta B)_X$ and $(\Delta B)_Y$ are both nonvanishing, the sum can be vanish if the first bracket in (92) cancels out. This happens if the X and Y particles have the same mass and $\Gamma_X = \Gamma_Y$.

4.3 Baryon number violation in Grand Unified Theories

The Grand Unified Theories (for a review, see [80]) try to describe the fundamental interactions by means of a unique gauge group G which contains the Standard Model (SM) gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The fundamental idea of GUTs is that at energies higher than a certain energy threshold $M_{\rm GUT}$ the group symmetry is G and that, at lower energies, the symmetry is broken down to the SM gauge symmetry, possibly through a chain of symmetry breakings

$$G \stackrel{M_{\rm GUT}}{\rightarrow} G_{(1)} \stackrel{M_1}{\rightarrow} G_{(2)} \stackrel{M_2}{\rightarrow} \cdots \stackrel{M_n}{\rightarrow} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{93}$$

corresponding to

$$M_{\rm GUT} > M_1 > M_2 > \cdots M_W, G \supset G_{(1)} \supset G_{(2)} \supset \cdots SU(3)_C \otimes SU(2)_L \otimes U(1)_Y. \tag{94}$$

What is the main motivation for invoking GUTs? Gauge couplings (couplings to gauge fields) are charactherized by a dimensionless constant g, or equivalently by $\alpha = g^2/4\pi$. (For electromagnetism, g is the electron charge and α evaluated at low energy is the fine structure constant $\alpha_{\rm em} = 1/137$.) Gauge couplings are not supposed to be extremely small, and one should take $g \sim 1$ for crude order of magnitude estimates (making α one or two orders of magnitude below 1). Assuming small couplings, the perturbative effects usually dominate, and we focus on them for the moment. With perturbative quantum effects included, the effective masses and couplings depend on the relevant energy scale Q. The dependence on Q (called 'running') can be calculated through the renormalization group equations (RGE's), and is logarithmic. In the context of collider physics, Q can be taken to be the collision energy, if there are no bigger relevant scales (particle masses). For the Standard Model there are three gauge couplings, α_i where $i = 3, 2, 1$, corresponding for respectively to the strong interaction (colour $SU(3)_C$) the non-abelian electroweak interaction ($SU(2)_L$) and electroweak hypercharge ($U(1)_Y$). (The electromagnetic gauge coupling is given by $\alpha^{-1} = \alpha_1^{-1} + \alpha_2^{-1}$.) In the one-loop approximation, ignoring the Higgs field, their running is given (at one loop) by

$$\frac{d\alpha_i}{d\ln(Q^2)} = \frac{b_i}{4\pi}\alpha_i^2\,. \tag{95}$$

The coefficients b_i depend on the number of particles with mass $\ll Q$. Including all particles in the minimal supersymmetric standard model gives $b_1 = 11$, $b_2 = 1$ and $b_3 = -3$.

Using the values of α_i measured by collider experiments at a scale $Q \simeq 100\,\text{MeV}$, one finds that all three couplings become equal at a scale [2, 39, 82][2] $Q = M_{\rm GUT}$, where

$$M_{\rm GUT} \simeq 2 \times 10^{16}\,\text{GeV}. \tag{96}$$

The unified value is

$$\alpha_{\rm GUT} \simeq 1/25. \tag{97}$$

One explanation of this remarkable experimental result may be that there is a GUT, involving a higher symmetry with a single gauge coupling, which is unbroken above the scale $M_{\rm GUT}$. Another might be that field theory becomes invalid above the unification scale, to be replaced by something like weakly coupled string theory or M-theory [124] which is the source of unification. At the time of writing there is no consensus about which explanation is correct, but in this section we will focus on Grand Unified Theories and their relevance for baryogenesis.

It is a general property of GUTs that the same representation may contain both quarks and leptons and therefore there exist gauge bosons which mediate gauge interactions among fermions having different baryon number. This is not enough –though– to conclude that in GUTs the baryon number is violated, because it might be possible to assign a baryonic charge to the gauge bosons in such a way that each vertex boson-fermion-fermion the baryon number is conserved. Let us discuss this crucial point in more detail.

The fundamental fermions of the SM are

$$\begin{aligned} \ell_L &= (1, 2, -1/2), \\ Q_L &= (3, 2, 1/6), \\ e^c_L &= (1, 1, 1), \\ u^c_L &= (\overline{3}, 1, -2/3), \\ d^c_L &= (\overline{3}, 1, 1/3), \end{aligned} \tag{98}$$

where in parenthesis we have written then $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers and all the spinors are left-handed. Given two spinors ψ_L and χ_L, it is possible to define a renormalizable coupling to a gauge boson V_μ by

$$i\,\psi^\dagger_L \sigma^\mu\,\chi_L\,V_\mu + \text{h.c.}, \tag{99}$$

where $\sigma^\mu = (\mathbf{1}, \vec{\sigma})$ and $\vec{\sigma}$ are the Pauli matrices. At this point one may try to write down all the couplings of the form (99) starting from the spinors of the SM and identify all the possible gauge bosons which may be present in a GUT having the same spinors of the SM. Of course, the same gauge boson may be coupled to more than one pair of spinors. If all the spinor pairs have the same baryon number B, then it suffices to assign a baryon number

[2]To be precise, $\frac{5}{3}\alpha_1 = \alpha_2 = \alpha_3 = \alpha_{\rm GUT}$, the factor 5/3 arising because the historical definition of α_1 is not very sensible. In passing we note that the unification fails by many standard deviations in the absence of supersymmetry, which may be construed as evidence for supersymmetry and anyhow highlights the remarkable accuracy of the experiments leading to this result.

$-B$ to the gauge boson and obtain a baryon number conserving theory. If there exist gauge bosons which couple to spinor pairs having different baryon number, one may write down baryon number violating interactions. These bosons are given in Table 1, where we have indicated, for every gauge boson, all the possible interactions and the corresponding baryon numbers B and baryon minus lepton numbers $B-L$

Table 1

Gauge boson	spinors	B	$B-L$
$V^1=(3,2,-5/6)$	$u_L^{c\dagger}Q_L$	2/3	2/3
	$Q_L^\dagger e_L^c$	-1/3	2/3
	$\ell_L^\dagger d_L^c$	-1/3	2/3
$V^2=(3,2,1/6)$	$\ell_L^\dagger u_L^c$	-1/3	2/3
	$d_L^{c\dagger}Q_L$	2/3	2/3

Of course, every gauge boson listed in Table 1 has the corresponding antiboson. One can repeat the same procedure to identify the scalar bosons S which may mediate baryon number violation interactions via fermions. The generic coupling reads

$$i\,\chi_L^T\,\sigma^2\,\psi_L\,S+\text{h.c.} \tag{100}$$

If we consider all the spinor pairs $\chi_L^T\psi_L$, even belonging to different families, we get the following possibilities

Table 2

Scalar boson	spinors	B	$B-L$
$S^1=(3,1,-1/3)$	$\ell^\dagger Q_L^\dagger$	-1/3	2/3
	Q_LQ_L	2/3	2/3
	$e_L^c u_L^c$	-1/3	2/3
	$d_L^{c\dagger}u_L^{c\dagger}$	2/3	2/3
$S^2=(3,1,-4/3)$	$e_L^c d_L^c$	-1/3	2/3
	$u_L^{c\dagger}u_L^{c\dagger}$	2/3	2/3
$S^3=(3,3,-1/3)$	$\ell_L^\dagger Q_L^\dagger$	-1/3	2/3
	Q_LQ_L	2/3	2/3

Out of all possible scalar and gauge bosons which may couple to the fermions of the SM, only the five that we have listed may give rise to interactions which violate the baryon number. A crucial point for what we will be discussing in the following is that each of these bosons have the same combination $B-L$, which means that this combination may be not violated in any vertex boson-fermion-fermion. This is quite a striking result and originates only from having required the invariance under the SM gauge group and that the only fermions of the theory are those of the SM.

The extension of the fermionic content of the theory may allow the presence of more heavy bosons which will possibly violate B and even $B-L$. In the Grand Unified Theories based on $SO(10)$ – for instance – there is another fermion which is a singlet under the SM gauge group and is identified with the antineutrino $N_L^c=(1,1,0)$. It carries lepton number equal to $L=-1$. It is possible to introduce a new scalar field S^4 which may couple to $N_L^c u_L^c$

and $d_L^{c\dagger} d_L^{c\dagger}$, thus violating the baryon number. It is remarkable that the choice for the lepton number of N_L^c leads to no new gauge boson which violates $B - L$. These considerations do not apply to supersymmetric models though (for a review see [56]). Indeed, for every fermionic degree of freedom there exist a superpartner (squark or slepton) which does have the same quantum number. Furthermore, in the Minimal Supersymmetric Standard Model (MSSM) one has to introduce two Higgs doublets $H_1 = (1, 2, -1/2)$ and $H_2 = (1, 2, 1/2)$ and the corresponding fermionic superpartners, the so-called higgsinos $\widetilde{H}_{1,2}$. Finally, every gauge boson has its own superparner, the gaugino. In this large zoo of new particles, one can easily find couplings that violate B and $B-L$. For instance, the higgsino $\widetilde{H}_1$ may couple to the quark doublet Q_L and to the scalars S^1 and S^3 of the Table 2. The pair $\widetilde{H}_1^\dagger Q_L^\dagger$ has baryon number $B = -1/3$ and $B-L = -1/3$ and both quantum numbers are not conserved. Nevertheless, in the supersymmetric models which are phenomenologically acceptable, even without considering the presence of superheavy particles, it is necessary to suppress some supersymmetric couplings which would lead at the weak scale to a proton decay at a rate which is too fast for being in agreement with the tight experimental constraints . One commonly accepted solution is to introduce a discrete symmetry Z_2, called R-parity, under which all the fields of the SM are even and all the superpartners are odd. The scalar component of any chiral supermultiplet has the following R-parity number

$$R = (-1)^{3(B-L)}, \tag{101}$$

while the corresponding fermion has the same number multiplied by -1. If we impose that R-parity is exact, then it is easy to check that, besides suppressing the fast proton decay at the weak scale, one avoids the presence B and L violating couplings of heavy fields with the light fermionic fields of the MSSM. Indeed, all the heavy bosons of Tables 1 and 2 have R-parity $R = 1$, while – for instance – the pair $\widetilde{H}_1^\dagger Q_L^\dagger$ has parity $R = -1$. Similar considerations apply to other fermionic pairs.

We conclude that in the GUTs, both supersymmetric and non-supersymmetric, no $B-L$ asymmetry may be generated through the out-of-equilibrium decay of gauge boson fields. We will mention in the following – though – that the generation of such an asymmetry is possible in the framework of (supersymmetric) $SO(10)$ via the out-of-equilibrium decay of the right-handed (s)neutrino, *i.e.* via the decay of a superheavy fermion (scalar).

After having learned that GUTs are the perfect arena for baryon number violating interactions, we will illustrate now some features of the out-of-equilibrium decay scenario within some specific GUTs, like $SU(5)$ and $SO(10)$.

4.3.1 The case of $SU(5)$

The gauge group $SU(5)$ is the smallest group containing the SM gauge group and as such it represents the most appealing candidate to build up a Grand Unified Theory. The non-supersymmetric version of $SU(5)$ is – however – already ruled out by its prediction of the proton lifetime $\tau_p \sim 10^{30}$ years, which is in disagreement with the experimental lower bound $\tau_p \gtrsim 10^{32}$ years [9]. Recent precise measurements of coupling constants at LEP suggest that the supersymmetric extension of $SU(5)$ gives a consistent picture of coupling unification [2, 39, 82] and is a viable possibility.

The fermionic content of $SU(5)$ is the same as the one in the SM. Therefore, as we explained in Section 4.3, it is not possible to create any asymmetry in $B-L$. Fermions are assigned to the reducible representation $\overline{5}_f \oplus 10_f$ as

$$\overline{5}_f = \{d_L^c, \ell_L\} \tag{102}$$

and

$$10_f = \{Q_L, u_L^c, e_L^c\}\,. \tag{103}$$

There are 24 gauge bosons which belong to the adjoint representation 24_V and may couple to the fermions through the couplings

$$\frac{g}{\sqrt{2}} 24_V \left[(\overline{5}_f)^\dagger\,(\overline{5}_f) + (10_f)^\dagger\,10_f\right]. \tag{104}$$

Among the 24 gauge bosons there are the bosons $XY = V^1 = (3, 2, -5/6)$ (and their CP-conjugate) which may decay violating the baryon number: $XY \to QL, \overline{QQ}$, where Q and L denotes an arbitrary quark and lepton, respectively. They have electric charges $Q_X = -1/3$ and $Q_Y = -4/3$. The mass and the couplings of these bosons are determined by the gauge coupling unification

$$\begin{aligned} M_{XY} &\simeq 5\times 10^{14}\,\text{GeV}, \quad \alpha_{\text{GUT}} \simeq 1/45, \quad \text{non}-\text{supersymmetric} \;\; SU(5), \\ M_{XY} &\simeq 10^{16}\,\text{GeV}, \quad \alpha_{\text{GUT}} \simeq 1/24, \quad \text{supersymmetric} \;\; SU(5). \end{aligned} \tag{105}$$

While in the gauge sector the structure is uniquely determined by the gauge group, in the Higgs sector the results depend upon the choice of the representation. The Higgs fields which couple to the fermions may be in the representation 5_H or in the representations 10_H, 15_H, 45_H and 50_H. If we consider the minimal choice 5_H, we obtain

$$h_U\,(10_f)^T\,(10_f)\,5_H + h_D\,(\overline{5}_f)^T\,(10_f)\,\overline{5}_H, \tag{106}$$

where $h_{U,D}$ are matrices in the flavor space. The representation 5_H contains the Higgs doublet of the SM, (1,2,1/2) and the triplet $S^1 = (3, 1, -1/3)$ which is B-violating. Unfortunately, this minimal choice of the Higgs sector does not suffice to explain the baryon number of the Universe. The CP violation is due to the complex phases which cannot be reabsorbed by field redifinition (they are physical) in the Yukawa sector. At the tree-level these phases do not give any contribution to the baryon asymmetry and at the one-loop level the asymmetry is proportional to

$$\text{Im Tr}\,\left(h_U^\dagger h_U h_D^\dagger h_D\right) = 0, \tag{107}$$

where the trace is over generation indices. This is because the Higgs on the external and internal legs of the one-loop interference diagrams is the same. A net baryon number only appears at three-loop, resulting in a baryon asymmetry $\sim 10^{-16}$ which is far too small to explain the observed one. The same problem in present in the supersymmetric version of $SU(5)$ where one has to introduce two Higgs supeerfields 5_H and $\overline{5}_{\overline{H}}$ [55].

The problem of too tiny CP violation in $SU(5)$ may be solved by complicating further the Higgs sector. One may introduce an extra scalar $5_H'$ with the same quantum numbers of 5_H, but with a different mass and/or lifetime [101]. In that case one-loop diagrams with exchange of $5'_H$ instead of 5_H can give rise to a net baryon number proportional

$$\text{Im Tr}\left(h'^{\dagger}_U h_U h'^{\dagger}_D h_D\right), \tag{108}$$

where $h'_{U,D}$ are the couplings of $5'_H$ to QL and $\overline{QQ}$, respectively. A second alternative is to introduce a different second Higgs representation. For example, adding a Higgs in the 45 representation of $SU(5)$ an adequate baryon asymmetry may be producedb for a wide range of the parameters [57].

4.3.2 The case of $SO(10)$

In the GUT based on $SO(10)$ the spontaneous breaking down to the SM gauge group is generally obtained through different steps (for a general review, see [117]). The main two channels are

$$\begin{aligned}
SO(10) &\overset{M_{\rm GUT}}{\rightarrow} G_{224} \overset{M_R}{\rightarrow} G_{214} \overset{M_C}{\rightarrow} G_{2113} \overset{M_{B-L}}{\rightarrow} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \\
SO(10) &\overset{M_{\rm GUT}}{\rightarrow} G_{224} \overset{M_C}{\rightarrow} G_{2213} \overset{M_R}{\rightarrow} G_{2113} \overset{M_{B-L}}{\rightarrow} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y,
\end{aligned} \tag{109}$$

where

$$\begin{aligned}
G_{224} &= SU(2)_L \otimes SU(2)_R \otimes SU(4), \\
G_{214} &= SU(2)_L \otimes U(1)_{I_{3R}} \otimes SU(4), \\
G_{2113} &= SU(2)_L \otimes U(1)_{I_{3R}} \otimes U(1)_{B-L} \otimes SU(3)_C, \\
G_{2213} &= SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes SU(3)_C,
\end{aligned} \tag{110}$$

where the four intermediate scales have not to be necessarily different from each other. We notice that,

– if we are interested in the generation of an asymmetry in $B-L$, the relevant scale is the scale at which the abelian group $U(1)_{B-L}$ breaks down, *i.e.* M_{B-L}, and not the Grand Unification scale $M_{\rm GUT}$;

– it is usually not possible to generate any baryon asymmetry at the scale $M_{\rm GUT}$. Indeed, the fermionic content of $SO(10)$ is the one of the SM plus a right-handed neutrino $N^c_L = (1, 1, 0)$. All the fermions belonging to the same generation are contained in the spinorial representation 16_f. Differently from what happens for the case of $SU(5)$, now all the fermions posses the corresponding antifermions and it is possible to define a conjugation operator of the charge C starting from the operators of $SO(10)$, in such a way that, if $SO(10)$ is not broken, then C is conserved [96]. In the simplest mechanism for the breaking of $SO(10)$, the one into G_{224}, a crucial role is played by the 54_H. In such a case there is a symmetry $SU(2)_L \otimes SU(2)_R$ [104, 93, 94] with equal coupling constants g_L and g_R and consequently, C is still a symmetry of the theory. It is possible to see that, with this choice of the Higgs representation, C is not broken until $U(1)_{B-L}$ is broken, *i.e.* at the scale M_{B-L} [3]. At this scale, the right-handed neutrino acquires a Majorana mass $M_N = \mathcal{O}(M_{B-L})$ and

[3] In fact, if one uses the 210_H to break $SO(10)$, one can maintain the gauge part of the left-right symmetry but not C. Th eimplications for baryogenesis are discussed in [23].

its out-of-equilibrium decays may generate a nonvanishing $B-L$ asymmetry [47]. We will return to this point later. With a more complicated choice of the Higgs representation it is possible to break C at the scale M_R where $SU(2)_R$ is broken and in such a case baryogenesis may take place at that scale.

5 The out-of-equilibrium decay scenario and the thermal history of the Universe

The out-of-equilibrium scenario that we have depicted in the previous section is operative only if a nonequilibrium number density of X heavy bosons was present in the early Universe. Usually massive particles are in equilibrium at at high temperatures, $T \gg M_X$ and their number density exceeds the equilibrium one when T becomes of the same order of the mass M_X. We have seen that, if the decay rate is small enough around $T \sim M_X$, see Eq. (58), then departure from equilibrium is attained and the subsequent decays of X and $\overline{X}$ particles may produce the observed baryon number asymmetry. The basic assumption – however – of this picture is that the superheavy bosons were as abundant as photons at very high temperatures $T \gtrsim M_X$.

If the X particles are gauge or Higgs bosons of Grand Unification, the situation is somewhat more complicated because they might have never been in thermal equilibrium at the very early stages of the evolution of the Universe. Even if the temperature of the primeval plasma was higher than the Grand Unified scale $M_{\rm GUT} \sim 10^{16}$ GeV, the rate of production of superheavy particles would be smaller than the expansion rate of the Universe and the number density of superheavy bosons could always be smaller than the equilibrium one. Secondly, the temperature of the Universe might be always smaller than $M_{\rm GUT}$ and correspondingly the thermally produced X bosons might be never as abundant as photons, making their role in baryogensis negligible. All these considerations depend crucially upon the thermal history of the Universe and deserve a closer look.

5.1 Inflation and reheating: the old days

The flatness and the horizon problems of the standard big bang cosmology are elegantly solved if during the evolution of the early Universe the energy density happened to be dominated by some form of vacuum energy and comoving scales grow quasi-exponentially [54]. An inflationary stage is also required to dilute any undesirable topological defects left as remnants after some phase transition taking place at early epochs.

The vacuum energy driving inflation is generally assumed to be associated to the potential $V(\phi)$ of some scalar field ϕ, the *inflaton*, which is initially displaced from the minimum of its potential. As a by-product, quantum fluctuations of the inflaton field may be the seeds for the generation of structure and the fluctuations observed in the cosmic microwave background radiation, $\delta T/T \sim 10^{-5}$ [83, 90, 84].

Inflation ended when the potential energy associated with the inflaton field became smaller than the kinetic energy of the field. By that time, any pre-inflation entropy in the Universe had been inflated away, and the energy of the universe was entirely in the form of coherent oscillations of the inflaton condensate around the minimum of its potential. The Universe may be said to be frozen after the end of inflation. We know that somehow the low-entropy cold Universe dominated by the energy of coherent motion of the ϕ field must be transformed into a high-entropy hot Universe dominated by radiation. The process by which the energy of the inflaton field is transferred from the inflaton field to radiation has been dubbed *reheating*. In the old theory of reheating [35, 1], the simplest way to envision this process is if the comoving energy density in the zero mode of the inflaton decays into normal particles, which then scatter and thermalize to form a thermal background. It is usually assumed that the decay width of this process is the same as the decay width of a free inflaton field.

Of particular interest is a quantity known as the reheat temperature, denoted as T_{RH}. The reheat temperature is calculated by assuming an instantaneous conversion of the energy density in the inflaton field into radiation when the decay width of the inflaton energy, Γ_ϕ, is equal to H, the expansion rate of the universe.

The reheat temperature is calculated quite easily. After inflation the inflaton field executes coherent oscillations about the minimum of the potential. Averaged over several oscillations, the coherent oscillation energy density redshifts as matter: $\rho_\phi \propto a^{-3}$, where a is the Robertson–Walker scale factor. If we denote as ρ_I and a_I the total inflaton energy density and the scale factor at the initiation of coherent oscillations, then the Hubble expansion rate as a function of a is

$$H^2(a) = \frac{8\pi}{3}\frac{\rho_I}{M_{\rm P}^2}\left(\frac{a_I}{a}\right)^3 . \tag{111}$$

Equating $H(a)$ and Γ_ϕ leads to an expression for a_I/a. Now if we assume that all available coherent energy density is instantaneously converted into radiation at this value of a_I/a, we can find the reheat temperature by setting the coherent energy density, $\rho_\phi = \rho_I(a_I/a)^3$, equal to the radiation energy density, $\rho_R = (\pi^2/30)g_* T_{RH}^4$, where g_* is the effective number of relativistic degrees of freedom at temperature T_{RH}. The result is

$$T_{RH} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_{\rm P}} \; = 0.2\left(\frac{200}{g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_{\rm P}} \; . \tag{112}$$

In the simplest chaotic inflation model, the inflaton potential is given by

$$V(\phi) = \frac{1}{2}M_\phi^2\phi^2, \tag{113}$$

with

$$M_\phi \sim 10^{13}\ \mathrm{GeV} \tag{114}$$

in order to reproduce the observed temperature anisotropies in the microwave background [90]. Writing $\Gamma_\phi = \alpha_\phi M_\phi$, one finds

$$T_{RH} \simeq 10^{15}\sqrt{\alpha_\phi}\ \mathrm{GeV}. \tag{115}$$

5.2 GUT baryogenesis and the old theory of reheating: a Herculean task

There are very good reasons to suspect that GUT baryogenesis is not in a good shape in the old theory of reheating.

5.2.1 Kinematical suppression of superheavy particles

The density and temperature fluctuations observed in the present universe, $\delta T/T \sim 10^{-5}$, require the inflaton potential to be extremely flat – that is $\alpha_\phi \ll 1$. This means that the couplings of the inflaton field to the other degrees of freedom cannot be too large, since large couplings would induce large loop corrections to the inflaton potential, spoiling its flatness. As a result, T_{RH} is expected to be much smaller than 10^{14}GeV by several orders of magnitude. As we have seen, the unification scale is generally assumed to be around 10^{16} GeV, and B-violating gauge bosons should have masses comparable to this scale. Baryon-number violating Higgs bosons may have a mass one or two orders of magnitude less. For example, in $SU(5)$ the B violating Higgs bosons in the five-dimensional representation that may have a mass as small as 10^{14} GeV. In fact, these Higgs bosons are more likely than gauge bosons to produce a baryon asymmetry since it is easier to arrange the requisite CP violation in the Higgs decay. But even the light B-violating Higgs bosons are expected to have masses larger than the inflaton mass, and it would be kinematically impossible to create them directly in ϕ decay, $\phi \to X\overline{X}$. This is because one expects

$$M_\phi \ll M_X. \tag{116}$$

5.2.2 Thermal production of heavy particles

One might think that the X bosons could be created by thermal scattering during the stage of thermalization of the decay products of the inflaton field. Indeed, the reheat temperature is best regarded as the temperature below which the Universe becomes radiation dominated. In this regard it has a limited meaning. For instance, it *should not* be interpretated as the maximum temperature obtained by the universe during reheating. The maximum temperature is, in fact, much larger than T_{RH}. One implication of this is that it is incorrect, to assume that the maximum abundance of a massive particle species X produced after inflation is suppressed by a factor of $\exp(-M_X/T_{RH})$ [26] and therefore it is incorrect to conclude that GUT baryogenesis is imcompatible with models of inflation where the reheating temperature is much smaller than the GUT scale and, in general, than the mass of the X particles, $T_{RH} \ll M_X$. Particles of mass much greater than the eventual reheating temperature T_{RH} may be created by the thermalized decay products of the inflaton. Indeed, a stable particle species X of mass M_X would be produced in the reheating process in sufficient abundance that its contribution to closure density today would be approximately $M_X^2\langle\sigma|v|\rangle(g_*/10^3)(M_X/10^4 T_{RH})^7$, where g_* is the number of effective degrees of freedom of the radiation energy density and $\langle\sigma|v|\rangle$ is the thermal average of the X annihilation cross section times the Møller flux factor. Thus, particles of mass as large as 10^4 times the reheating temperature may be produced in interesting abundance [26]. The number density n_X of particles X after freeze out and reheating may be easily computed [26]

$$\frac{n_X}{n_\gamma} \simeq 3\times 10^{-4}\left(\frac{100}{g_*}\right)^{3/2}\left(\frac{T_{RH}}{M_X}\right)^7\left(\frac{M_{\rm P}}{M_X}\right) \tag{117}$$

and is not exponentially suppressed. It is easy to check that for such small values of T_{RH}, the ratio (117) is always much larger than the equilibrium value

$$\left(\frac{n_X}{n_\gamma}\right)_{\rm EQ} = \left(\frac{\pi^{1/2}}{\xi(3)}\right)\left(\frac{M_X}{2T_{RH}}\right)^{3/2} e^{\frac{-M_X}{T_{RH}}}. \qquad (118)$$

This result is crucial for the out-of-equilibrium decay scenarios of baryogenesis. For instance, as we shall see, in theories where $B-L$ is a spontaneously broken local symmetry, as suggested by $SO(10)$ unification, the cosmological baryon asymmetry can be generated by the out-of-equilibrium decay of the lightest heavy Majorana right-handed neutrino N_1^c, whose typical mass is about 10^{10} GeV [47]. For reheat temperatures of the order of 10^9 GeV, the number density of the right-handed neutrino is about $3\times 10^{-2}\, n_\gamma$ and one can estimate the final bayon number to be of the order of $B\sim (n_{N_1^c}/n_\gamma)(\epsilon/g_*)\simeq 10^{-4}\epsilon$, where ϵ is the coefficient containing one-loop suppression factor and CP violating phases. The observed value of the baryon asymmetry, $B\sim 10^{-10}$, is then obtained without any fine tuning of parameters.

Exercise 2

Compute the maximum temperature during the process of reheating. *Hint*: Consider the early-time solution for radiation (*i.e.* when $H\gg\Gamma_\Phi$ and before a significant fraction of the comoving coherent energy density is converted to radiation).

5.2.3 The gravitino problem

There is one more problem associated with GUT baryogenesis in the old theory of reheating, namely the problem of relic gravitinos [41]. If one has to invoke supersymmetry to preserve the flatness of the inflaton potential, it is mandatory to consider the cosmological implications of the gravitino – a spin-(3/2) particle which appears in the extension of global supersymmetry to local supersymmetry – or supergravity [50]. The gravitino is the fermionic superpartner of the graviton and has interaction strength with the observable sector – that is the SM particles and their superpartners – inversely proportional to the Planck mass. One usually associates the scale of supersymmetry breaking with the electroweak scale in order to handle the hierarchy problem [103] and the mass of the gravitino is of order of the weak scale, $m_{3/2}=\mathcal{O}(1)$ TeV. The decay rate of the gravitino is given by

$$\Gamma_{3/2}\sim\frac{m_{3/2}^3}{M_{\rm P}^2}\sim (10^5\ {\rm sec})^{-1}\left(\frac{m_{3/2}}{\rm TeV}\right)^3. \qquad (119)$$

The slow decay rate of the gravitinos is the essential source of the cosmological problems because the decay products of the gravitino will destroy the ^{4}He and D nuclei by photodissociation, and thus successful nucleosynthesis predictions. The most stringent bound comes from the resulting overproduction of D + ^{3}He, which would require that the gravitino abundance is smaller than $\sim 10^{-10}$ relative to the entropy density at the time of reheating after inflation [60]

$$\frac{n_{3/2}}{s}\lesssim (10^{-10}-10^{-11}). \qquad (120)$$

The Boltzmann equation governing the number density of gravitinos $n_{3/2}$ during the thermalization stage after inflation is

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} \simeq \langle \Sigma_{\rm tot} v \rangle n^2_{\rm light}, \tag{121}$$

where $\Sigma_{\rm tot} \propto 1/M_{\rm P}^2$ is the total cross section determining the rate of production of gravitinos and $n_{\rm light} \sim T^3$ represents the number density of light particles in the thermal bath. The number density of gravitinos at thermalization is readily obtained solving Eq. (121) and reads

$$\frac{n_{3/2}}{s} \simeq 10^{-2}\,\frac{T_{RH}}{M_{\rm P}}. \tag{122}$$

Comparing Eqs. (120) and (122), one may obtain an upper bound on the reheating temperature after inflation

$$T_{RH} \lesssim (10^{10} - 10^{11})\ {\rm GeV}. \tag{123}$$

Therefore, if $T_{RH} \sim M_{\rm GUT}$, gravitinos would be abundant during nucleosynthesis and destroy the good agreement of the theory with observations. However, if the initial state after inflation was free from gravitinos, the reheating temperature seems to be too low to create superheavy X bosons that eventually decay and produce the baryon asymmetry – even taking into account the previous considerations about the fact that the maximum temperature during reheating is not T_{RH} [73, 26].

5.3 Inflation and reheating: the new wisdom

The outlook for GUT baryogenesis has brightened recently with the realization that reheating may differ significantly from the simple picture described above [68, 63, 64, 65, 66]. In the first stage of reheating, called *preheating* [68], nonlinear quantum effects may lead to an extremely effective dissipational dynamics and explosive particle production even when single particle decay is kinematically forbidden. Particles can be produced in the regime of a broad parametric resonance, and it is possible that a significant fraction of the energy stored in the form of coherent inflaton oscillations at the end of inflation is released after only a dozen or so oscillation periods of the inflaton. What is most relevant for these lectures is that preheating may play an extremely important role for baryogenesis [74, 3, 75] and, in particular, for GUT generation of the baryon asymmetry. Indeed, it was shown in [74, 75] that the baryon asymmetry can be produced efficiently just after the preheating era, thus solving many of the problems that GUT baryogenesis had to face in the old picture of reheating.

The presence of a preheating stage at the beginning of the reheating process is based on the fact that, for some parameter ranges, there is a new decay channel that is non-perturbative: due to the coherent oscillations of the inflaton field stimulated emissions of bosonic particles into energy bands with large occupancy numbers are induced [68]. The modes in these bands can be understood as Bose condensates, and they behave like classical waves. The back-reaction of these modes on the homogeneous inflaton field and the rescattering among themselves produce a state that is far from thermal equilibrium and may induce very interesting phenomena, such as non-thermal phase transitions [69, 121, 112] with production of a stochastic background of gravitational waves [66] and of heavy particles in a state far from equilibrium, which may constitute today the dark matter in our Universe [24, 25].

The idea of preheating is relatively simple, the oscillations of the inflaton field induce mixing of positive and negative frequencies in the quantum state of the field it couples to because of the *time-dependent* mass of the quantum field. Let us focus – for sake of simplicity – to the case of chaotic inflation, with a massive inflaton ϕ with quadratic potential $V(\phi) = \frac{1}{2}M_\phi^2\phi^2$, $M_\phi \sim 10^{13}$ GeV, and coupled to a massless scalar field χ via the quartic coupling $g^2\phi^2\chi^2$.

The evolution equation for the Fourier modes of the χ field with momentum k is

$$\ddot{X}_k + \omega_k^2 X_k = 0, \tag{124}$$

with

$$\begin{aligned} X_k &= a^{3/2}(t)\chi_k, \\ \omega_k^2 &= k^2/a^2(t) + g^2\phi^2(t). \end{aligned} \tag{125}$$

This Klein-Gordon equation may be cast in the form of a Mathieu equation

$$X_k'' + [A(k) - 2q\cos 2z]X_k = 0, \tag{126}$$

where $z = M_\phi t$ and

$$\begin{aligned} A(k) &= \frac{k^2}{a^2M_\phi^2} + 2q, \\ q &= g^2\frac{\Phi^2}{4M_\phi^2}, \end{aligned} \tag{127}$$

where Φ is the amplitude and M_ϕ is the frequency of inflaton oscillations, $\phi(t) = \Phi(t)\sin(M_\phi t)$. Notice that, at least initially when $\Phi = cM_{\rm P} \lesssim M_{\rm P}$

$$g^2\frac{\Phi^2}{4M_\phi^2} \sim g^2\, c^2\frac{M_{\rm P}^2}{M_\phi^2} \sim g^2\, c^2 \times 10^{12} \gg 1 \tag{128}$$

and the resonance is broad. For certain values of the parameters (A, q) there are exact solutions X_k and the corresponding number density n_k that grow exponentially with time because they belong to an instability band of the Mathieu equation (for a recent comprehensive review on preheating after chaotic inflation [70] and references therein)

$$X_k \propto e^{\mu_k M_\phi t} \Rightarrow n_k \propto e^{2\mu_k M_\phi t}, \tag{129}$$

where the parameter μ_k depends upon the instability band and, in the broad resonance case, $q \gg 1$, it is ~ 0.2.

These instabilities can be interpreted as coherent "particle" production with large occupancy numbers. One way of understanding this phenomenon is to consider the energy of these modes as that of a harmonic oscillator, $E_k = |\dot{X}_k|^2/2 + \omega_k^2|X_k|^2/2 = \omega_k(n_k + 1/2)$. The occupancy number of level k can grow exponentially fast, $n_k \sim \exp(2\mu_k M_\phi t) \gg 1$, and these modes soon behave like classical waves. The parameter q during preheating determines the strength of the resonance. It is possible that the model parameters are such that parametric resonance does *not* occur, and then the usual perturbative approach would follow, with decay rate Γ_ϕ. In fact, as the Universe expands, the growth of the scale factor and the decrease of the amplitude of inflaton oscillations shifts the values of (A, q) along the stability/instability chart of the Mathieu equation, going from broad resonance, for $q \gg 1$, to narrow resonance, $q \ll 1$, and finally to the perturbative decay of the inflaton.

It is important to notice that, after the short period of preheating, the Universe is likely to enter a long period of matter domination where the biggest contribution to the energy density of the Universe is provided by the residual small amplitude oscillations of the classical inflaton field and/or by the inflaton quanta produced during the back-reaction processes. This period will end when the age of the Universe becomes of the order of the perturbative lifetime of the inflaton field, $t \sim \Gamma_\phi^{-1}$. At this point, the Universe will be reheated up to a temperature T_{RH} given in (112) obtained applying the old theory of reheating described in the previous section.

5.4 GUT baryogenesis and preheating

A crucial observation for baryogenesis is that even particles with mass larger than that of the inflaton may be produced during preheating. To see how this might work, let us assume that the interaction term between the superheavy bosons and the inflaton field is of the type $g^2\phi^2|X|^2$. During preheating, quantum fluctuations of the X field with momentum $\vec{k}$ approximately obey the Mathieu equation where now

$$A(k) = \frac{k^2 + M_X^2}{M_\phi^2} + 2q. \tag{130}$$

Particle production occurs above the line $A = 2q$. The width of the instability strip scales as $q^{1/2}$ for large q, independent of the X mass. The condition for broad resonance [68, 74]

$$A - 2q \lesssim q^{1/2} \tag{131}$$

becomes

$$\frac{k^2 + M_X^2}{M_\phi^2} \lesssim g\frac{\Phi}{M_\phi}, \tag{132}$$

which yields for the typical energy of X bosons produced in preheating

$$E_X^2 = k^2 + M_X^2 \lesssim g\Phi M_\phi, \tag{133}$$

By the time the resonance develops to the full strength, $\Phi^2 \sim 10^{-5} M_{\rm P}^2$. The resulting estimate for the typical energy of particles at the end of the broad resonance regime for $M_\phi \sim 10^{13}$ GeV is

$$E_X \sim 10^{-1} g^{1/2} \sqrt{M_\phi M_{\rm P}} \sim g^{1/2} 10^{15} \text{ GeV}. \tag{134}$$

Supermassive X bosons can be produced by the broad parametric resonance for $E_X > M_X$, which leads to the estimate that X production will be possible if $M_X < g^{1/2} 10^{15}$ GeV.

For $g^2 \sim 1$ one would have copious production of X particles (in this regime the problem is non-linear from the beginning and therefore $g^2 = 1$ has to be understood as a rough estimate of the limiting case) as heavy as 10^{15}GeV, i.e., 100 times greater than the inflaton mass. The only problem here is that for large coupling g, radiative corrections to the effective potential of the inflaton field may modify its shape at $\phi \sim M_{\rm Pl}$. However, this problem does not appear if the flatness of the inflaton potential is protected by supersymmetry.

This is a significant departure from the old constraints of reheating. Production of X bosons in the old reheating picture was kinematically forbidden if $M_\phi < M_X$, while in the new scenario it is possible because of coherent effects. It is also important to note that the particles are produced out-of-equilibrium, thus satisfying one of the basic requirements to produce the baryon asymmetry [114].

Scattering of X fluctuations off the zero mode of the inflaton field limits the maximum magnitude of X fluctuations to be $\langle X^2\rangle_{\rm max} \approx M_\phi^2/g^2$ [65]. For example, $\langle X^2\rangle_{\rm max} \sim 10^{-10}M_{\rm P}^2$ in the case $M_X = 10\, M_\phi$. This restricts the corresponding number density of created X-particles.

A potentially important dynamical effect is that the parametric resonance is efficient only if the self-interaction couplings of the superheavy particles are not too large. Indeed, a self-interaction term of the type $\lambda|X|^4$ provides a non-thermal mass to the X boson of the order of $(\lambda\langle X^2\rangle)^{1/2}$, but this contribution is smaller than the bare mass M_X, if $\lambda \lesssim g^2 M_X^2/M_\phi^2$. Self-interactions may also terminate the resonance effect because scattering induced by the coupling λ may remove particles from the resonance shells and redistribute their momenta [68]. But this only happens if, again, $\lambda \gg g^2$ [64].

The parametric resonance is also rendered less efficient when the X particles have a (large) decay width Γ_X, which is essential for the out-of-equilibrium decay to take place. Roughly speaking, one expects that the explosive production of particles takes place only if the typical time, τ_e, during which the number of X bosons grows by a factor of e, is smaller than the decay lifetime $\tau_X = \Gamma_X^{-1}$. During the broad resonance regime, typically $\tau_e \lesssim 10\, M_\phi^{-1}$. If we write the decay width by $\Gamma_X = \alpha_X\, M_X$, this requires $\alpha_X \lesssim 0.1 M_\phi/M_X$. Notice that smaller values of Γ_X are favored not only because particle production is made easier, but also because the superheavy particles may remain out-of-equilibrium for longer times, thus enhancing the final baryon asymmetry.

Using the methods developed in Refs. [63, 64, 65], one can study numerically the production of massive, unstable X particles in the process of the inflation decay [75]. Let us consider a model in which the oscillating inflaton field ϕ interacts with a scalar field X whose decays violate baryon number B. As we have learned, the simplest possibility for the X-particle is the Higgs field in the five-dimensional representation of $SU(5)$. We assume standard kinetic terms, minimal coupling with gravity, and a very simple potential for the fields of the form

$$V(\phi, X) = \frac{1}{2}M_\phi^2\phi^2 + \frac{1}{2}M_X^2 X^2 + \frac{1}{2}g^2\phi^2 X^2. \tag{135}$$

A fundamental parameter in GUT baryogenesis is n_X, the number density of the supermassive leptoquarks whose decays produce the baryon asymmetry. It will depend upon the value of Γ and q.

Since the supermassive bosons are more massive than the inflaton, one expects small kinetic energy in the excitations of the X field. From the potential of Eq. (135), the square

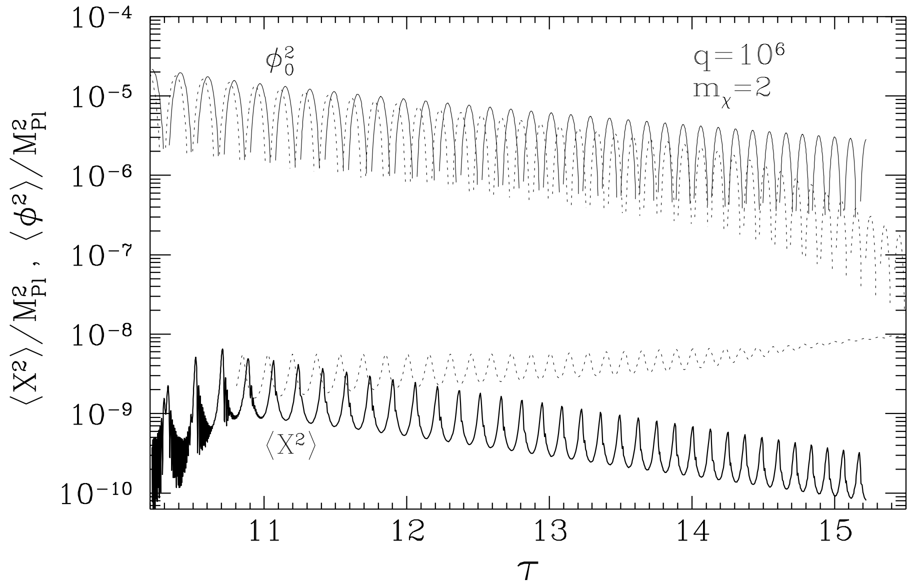


Figure 4: The variance of X with model parameters $q = 10^6$, $m_\chi = 2$, and $\Gamma \equiv \Gamma_X/M_\phi = 6\times10^{-2}$ is shown by the lower solid curve as a function of time. The upper solid curve corresponds to the inflaton zero mode. The dotted curves represent the same quantities for $\Gamma = 0$.

of the effective mass of the X field is

$$(M_X^{\mathrm{EFF}})^2 = M_X^2 + g^2\langle\phi^2\rangle \tag{136}$$

and the energy density in the X field will be

$$\rho_X \simeq (M_X^2 + g^2\langle\phi^2\rangle)\langle X^2\rangle. \tag{137}$$

Writing $\langle\phi^2\rangle$ as $\phi_0^2 + \langle\delta\phi^2\rangle$, one can define an analog of the X-particle number density as

$$n_X = \rho_X/M_X^{\mathrm{EFF}} = \left[4q(\phi_0^2 + \langle\delta\phi^2\rangle)/\phi_0^2(0) + m_\chi^2\right]^{1/2} M_\phi\langle X^2\rangle, \tag{138}$$

where $m_\chi = M_X/M_\phi$.

Eq. (138) enables one to calculate the number density of the created X-particles if the variances of the fields, $\langle X^2\rangle$, $\langle\delta\phi^2\rangle$, and the inflaton zero mode $\phi_0(\tau)$ (here t and τ are related by $M_\phi dt = a(\tau)d\tau$) are known.

The time evolution of the variance, $\langle X^2\rangle$, and of the inflaton zero mode, $\langle\phi\rangle$, is shown in Fig. 4, by the solid curves for the case $q = 10^6$, $m_\chi = 2$, and $\Gamma = 6\times10^{-2}$. We see that the particle creation reaches a maximum at $\tau \approx 10.8$ when $\langle X^2\rangle \approx 10^{-9}$ in the "valleys" between the peaks. At later times, $\tau > 10.8$, particle creation by the oscillating inflaton field can no longer compete with X-decays due to the non-zero value of Γ. For comparison, we show in the same figure the case $\Gamma = 0$ represented by the dotted curves [65]. In the $\Gamma = 0$ case, particle creation is able to compete with the expansion of the universe so that $\langle X^2\rangle$ remains roughly constant.

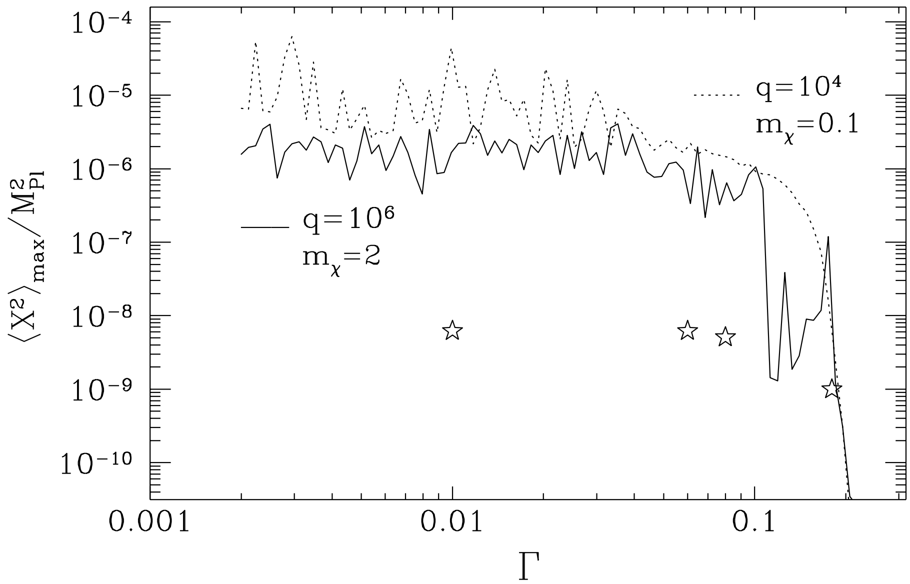


Figure 5: The maximum value of the variance of the X-field, $\langle X^2_{\rm max}\rangle$, is shown as function of Γ. Stars mark $\langle X^2_{\rm max}\rangle$ obtained in the full non-linear problem. $\langle X^2_{\rm max}\rangle$ in the Hartree approximation is shown by the dotted curve for $q = 10^4$, $m_\chi = 0.1$, and by the solid curve for $q = 10^6$, $m_\chi = 2$.

Using Eq. (138), one finds for the maximum number density of created X-particles

$$n_X = \left[4q\phi_0^2(10.8)/\phi_0^2(0) + m_\chi^2\right]^{1/2} M_\phi \langle X^2\rangle \approx \left[10^3 + m_\chi^2\right]^{1/2} M_\phi\langle X^2\rangle \approx 30 M_\phi\langle X^2\rangle. \quad (139)$$

It is easy to understand that if we increase the value of Γ, the parametric resonance will not be able to compete with the decay of X at earlier times. Moreover, for sufficiently large values of Γ, the resonance will be shut off in the linear regime.

In exploration of parameter space it turns out more convenient to go to the Hartree approximation which requires much less computing resources. The maximum value of the variance of X reached during the time evolution of the fields in the Hartree approximation is shown in Fig. 5 as a function of the parameters of the model. Here the stars also show the maximum of $\langle X^2(\tau)\rangle$ in the full non-linear problem for a few values of Γ. At small Γ the Hartree approximation overestimates $\langle X^2\rangle$ significantly [64, 65]. Nonetheless, at large values of Γ it is a quite reliable approach. One may see that $\langle X^2\rangle$ drops sharply when $\Gamma > 0.2$, and this critical value of Γ does not depend significantly upon m_X or q [75].

The most relevant case with $q = 10^8$, where X-bosons as massive as ten times the inflaton mass can be created, is shown in Fig. 6 in the Hartree approximation. Note, that two lower curves which correspond to Γ equal to 0.08 and 0.12 never reach the limiting value $\langle X^2\rangle_{\rm max} \sim 10^{-10} M_{\rm P}^2$, which is imposed by rescattering [65], and the Hartree approximation ought to be reliable in this cases.

As outlined above, one may consider a three part reheating process, with initial conditions corresponding to the frozen universe at the end of inflation. The first stage is

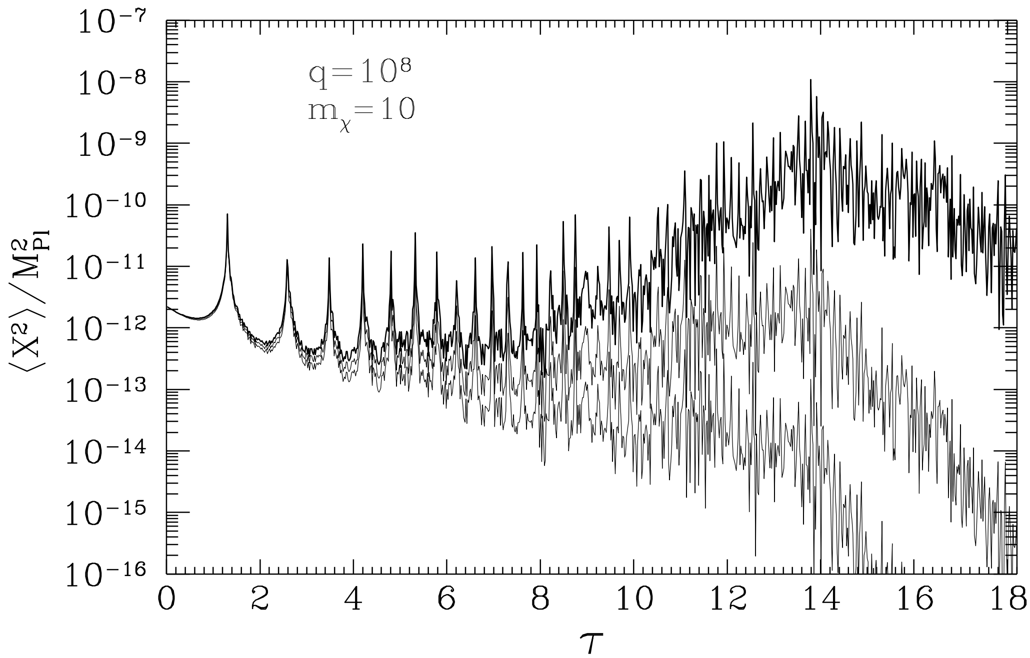


Figure 6: The time dependence of the variance of X in the Hartree approximation with model parameters $q = 10^8$, $m_\chi = 10$ and for three values of Γ, from top to bottom: 0.04, 0.08, 0.12.

explosive particle production, where a fraction δ of the energy density at the end of preheating is transferred to X bosons, with $(1-\delta)$ of the initial energy remaining in ϕ coherent oscillation energy. We assume that this stage occurs within a few Hubble times of the end of inflation. The second stage is the X decay and subsequent thermalization of the decay products. We assume that decay of an X–$\overline{X}$ pair produces a net baryon number ϵ, as well as entropy. Reheating is brought to a close in the third phase when the remaining energy density in ϕ oscillations is transferred to radiation.

The final baryon asymmetry depends linearly upon the ratio δ between the energy stored in the X particles at the end of the preheating stage and the energy stored in the inflaton field at the beginning of the preheating era [74]. The description simplifies if we assume zero initial kinetic energy of the Xs. One may also a assume that there are fast interactions that thermalize the massless decay products of the X. Then in a co-moving volume a^3, the total number of X bosons, $N_X = n_X a^3$, the total baryon number, $N_B = n_B a^3$, and the dimensionless radiation energy, $R = \rho_R a^4$, evolve according to

$$\begin{aligned} \dot{N}_X &= -\Gamma_X \left(N_X - N_X^{\rm EQ}\right); \quad \dot{R} = -aM_X \dot{N}_X; \\ \dot{N}_B &= -\epsilon \dot{N}_X - \Gamma_X N_B \left(N_X^{\rm EQ}/N_0\right). \end{aligned} \tag{140}$$

$N_X^{\rm EQ}$ is the total number of Xs in thermal equilibrium at temperature $T \propto R^{1/4}$, and N_0 is the equilibrium number of a massless degree of freedom in a comoving volume.

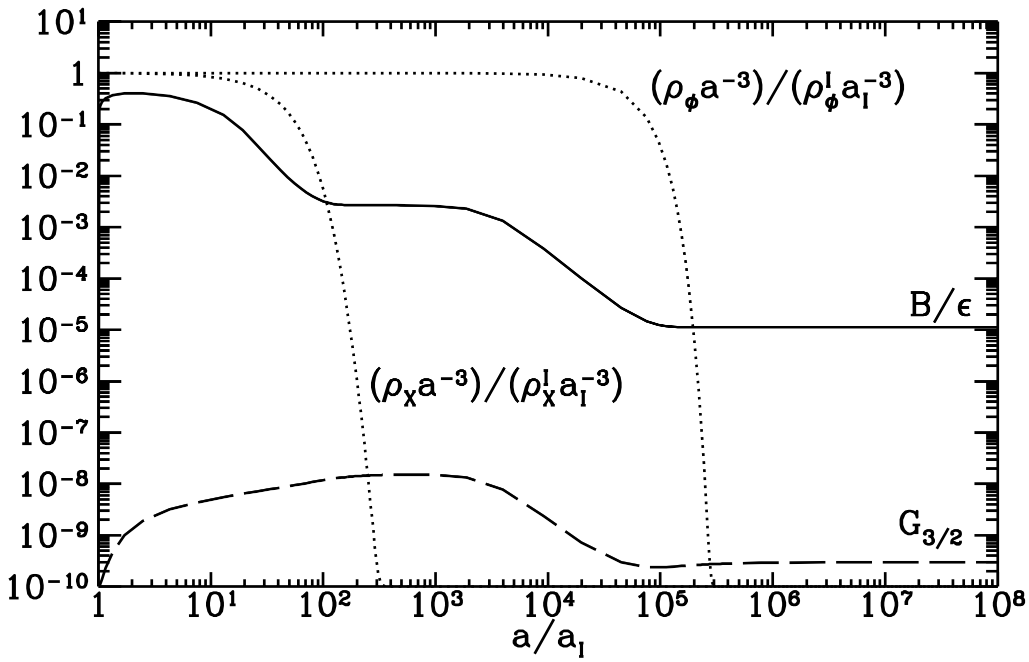


Figure 7: The evolution of the baryon number, the X number density, the energy density in ϕ oscillations, and the gravitino-to-entropy ratio as a function of the scale factor a.

Fig. 7 shows the results of an integration of Eqs. (140) in a toy model with $M_\phi = 10^{13}$GeV, $M_X = 10^{14}$GeV, $\Gamma_X = 5 \times 10^{-6} M_X$, $\Gamma_\phi = 5 \times 10^{-10} M_\phi$, and two degrees of freedom (b and $\overline{b}$). Initial conditions were chosen at $a = a_I$ to be $\rho_X = \rho_\phi \sim 10^{-4} M_\phi^2 M_{\rm P}^2$, and $R = N_B = 0$. The $\rho_X = \rho_\phi$ assumption corresponds to $\delta = 1/2$. The baryon number $B = n_B/s$ rapidly rises. However B decreases as entropy is created and X inverse reactions damp the baryon asymmetry. After most of the energy is extracted from the initial X background, the baryon number is further damped as entropy is created during the decay of energy in the ϕ background. One can also numerically integrate the equation governing the number density of gravitinos $n_{3/2}$. The result for $G_{3/2} = n_{3/2}/s$ is shown in Fig. 7. Notice that, even though gravitinos are copiously produced at early stages by scatterings of the decay products of the X, $G_{3/2}$ decreases as entropy is created during the subsequent decay of energy in the ϕ background.

Since the number of X bosons produced is proportional to δ, the final asymmetry is proportional to δ and $B/\epsilon \sim 10^{-9}$ can be obtained for δ as small as 10^{-6}. One can estimate this ratio as

$$\delta \simeq 3 \times 10^6 \sqrt{\frac{q}{10^6}}\, m_\chi \, \frac{\langle X^2 \rangle}{M_{\rm P}^2}. \tag{141}$$

Therefore, for $q = 10^8$ and $m_\chi = 10$, δ is of the order of $3 \times 10^8 \langle X^2 \rangle / M_{\rm P}^2$. Since the final baryon asymmetry scales approximately as Γ^{-1} and is given by $B \simeq 5 \times 10^{-4} \delta \epsilon (\Gamma / 5 \times 10^{-5})^{-1}$ [74], where ϵ is an overall parameter accounting for CP violation, one can see that the observed baryon asymmetry $B \simeq 4 \times 10^{-11}$ may be explained by the phenomenon of GUT baryogenesis after preheating if

$$\frac{\langle X^2\rangle}{M_{\rm P}^2}\simeq 5\times 10^{-13}\left(\frac{10^{-2}}{\epsilon}\right)\left(\frac{\Gamma}{5\times 10^{-5}}\right). \tag{142}$$

From Fig. 6 we can read that this only may happen if

$$\Gamma_X \lesssim 10^{-3}\, M_X. \tag{143}$$

This result may be considered very comfortable since we can conclude that whenever the resonance develops, i.e., when $\Gamma_X \lesssim 10^{-1} M_\phi = 10^{-2} M_X$, GUT baryogenesis after preheating is so efficient that the right amount of baryon asymmetry is produced for almost the entire range of values of the decay rate Γ_X. In other words, provided that superheavy X-bosons are produced during the preheating stage, they will be *ineffective* in producing the baryon asymmetry *only* if their decay rate falls in the range $10^{-3} M_X \lesssim \Gamma_X \lesssim 10^{-2} M_X$. GUT baryogenesis after preheating solves many of the serious drawbacks of GUT baryogenesis in the old theory of reheating where the production of superheavy states after inflation was kinematically impossible. Moreover, the out-of-equilibrium condition is naturally attained in our scenario since the distribution function of the X-quanta generated at the resonance is far from a thermal distribution. This situation is considerably different from the one present in the GUT thermal scenario where superheavy particles usually decouple from the thermal bath when still relativistic and then decay producing the baryon asymmetry. It is quite intriguing that out of all possible ways the parametric resonance may develop, Nature might have chosen only those ways without instantaneous thermalization and also with a successful baryogenesis scenario.

6 The baryon number violation in the Standard Model

In this section we will be concerned with the violation of the baryon and lepton number in the SM. It is well-known that by considering the most general Lagrangian invariant under the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ and assuming that the Higgs fields are color singlets, the Lagrangian is automatically invariant under global abelian symmetries which may be identified with the baryonic and leptonic symmetries. They are – therefore – accidental symmetries. As a result, it is not possible to violate B and L at the tree-level and at any order of perturbation theory: the proton is stable in the SM and any perturbative process which violates B and/or L in Grand Unified Theories is necessarily suppressed by powers of $M_{\rm GUT}/M_W$. Nevertheless, in many cases the perturbative expansion does not describe all the dynamics of the theory and – indeed – in 1976 't Hooft [120] realized that nonperturbative effects (instantons) may give rise to processes which violate the combination $B+L$, but not the orthogonal combination $B-L$. The probability of these processes to occur is exponentially suppressed, $\sim \exp(-4\pi/\alpha_W) \sim 10^{-150}$ where $\alpha_W = g_2^2/4\pi$ is the weak gauge coupling, and probably irrelevant today. In more extreme situations – like the primordial Universe at very high temperatures [34, 67, 76] – baryon and lepton number violation processes may be fast enough to play a significant role in baryogenesis. This will be the subject of the present section.

6.1 The $B + L$ anomaly

The violation of the baryonic number within the SM is due to the fact that the current corresponding to the global abelian group U_{B+L} – even though it is conserved at the classical level – is not conserved at the quantum level, that is the U_{B+L} is *anomalous*. Let us consider in the euclidean space the generating function $\exp[-Z] = \int \mathcal{D}\psi\,\mathcal{D}\overline{\psi}\exp[-S]$ [48]. The most general phase transformation onto the Dirac field ψ with mass m

$$\psi(x) \to e^{i(a+b\,\gamma_5)\theta(x)}\,\psi(x) \tag{144}$$

induces an additional term in the action given by

$$\delta S_0 = -\int d^4x\,\left[\overline{\psi}\,m\,\left(e^{2ib\gamma_5\theta(x)} - 1\right)\,\psi + \overline{\psi}\gamma^\mu(a + b\gamma_5)\psi\partial_\mu\theta(x)\right]. \tag{145}$$

The rotation (144) gives rise to a nontrivial jacobian due to the noninvariance of the measure $\mathcal{D}\psi\,\mathcal{D}\overline{\psi}$. This may be expressed as an additional contribution to the action

$$\delta S_1 = i\int d^4x\,\theta(x)\left[\frac{(a-b)}{8\pi^2}\,\mathrm{Tr}\,F^{(L)\mu\nu}\widetilde{F}^{(L)}_{\mu\nu} - \frac{(a+b)}{8\pi^2}\,\mathrm{Tr}\,F^{(R)\mu\nu}\widetilde{F}^{(R)}_{\mu\nu}\right], \tag{146}$$

where $F^{(L)\mu\nu}$ ($F^{(R)\mu\nu}$) is the field strength which couples to the left-handed (right-handed) current of the field ψ, while $\widetilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}$. Notice that we have absorbed the gauge coupling into the definitions of $F^{(L,R)\mu\nu}$ and the traces are over the group indices.

If the rotation (144) corresponds to the baryon number rotation, then

$$a = \frac{1}{3}\quad\text{and}\quad b = 0. \tag{147}$$

Integrating by parts (145) and requiring that the generating function is invariant under the baryonic number transformation, we obtain that the baryonic current $J^\mu_B = \sum_q \frac{1}{3}\overline{q}\gamma^\mu q$ is anomalous

$$\partial_\mu J^\mu_B = i\frac{N_F}{32\pi^2}\left(-g_2^2F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right), \tag{148}$$

where N_F is the number of fermionic families, $F^a_{\mu\nu}$ is the field strength of $SU(2)_L$ and $f_{\mu\nu}$ that of $U(1)_Y$ with coupling constants g_2 and g_1, respectively, and we have made use of the fact that $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta_{ab}$ for the $SU(2)_L$ generators and of the values $Y = 1/6,\ 2/3,\ -1/3$ for Q_L, u_R and d_R, respectively.

Analogously, if we consider the rotation associated to the lepton number

$$a = 1\quad\text{and}\quad b = 0, \tag{149}$$

we obtain

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L, \tag{150}$$

where $J^\mu_L = \sum_\ell(\overline{\ell}\gamma^\mu\ell + \overline{\nu}_\ell\gamma^\mu\nu_\ell)$ is the leptonic current. The relation (149) shows explicitly that the current associated to $B - L$ is conserved. In fact – since each quark and leton family gives the same contribution to the anomaly and each leptonic flavor is conserved in the SM at the classical level – the conserved charges are actually three

$$\ell_i \equiv \frac{1}{3}B - L_i, \tag{151}$$

where L_i ($i = e,\ \mu,\ \tau$) are the leptonic flavors and $\sum_i L_i = L$.

6.2 Topology of $SU(2)_L$ and baryon number violation

In the previous subsection we have described how the chiral anomaly induces the nonconservation of the baryonic current. We now wish to understand what is the physical significance of the terms in the righ-hand side of the Eq. (148). First, we note that these terms may be reexpressed as

$$\partial_\mu J^\mu_B = i\frac{N_F}{32\pi^2}\left(-g_2^2\partial_\mu K^\mu + g_1^2\partial_\mu k^\mu\right), \tag{152}$$

where

$$\begin{aligned} K^\mu &= 2\epsilon^{\mu\nu\rho\sigma}\left(\partial_\nu A^a_\rho A^a_\sigma - \frac{1}{3}g_2\epsilon_{abc}A^a_\nu A^b_\rho A^c_\sigma\right), \\ k^\mu &= 2\epsilon^{\mu\nu\rho\sigma}\left(\partial_\nu B_\rho B_\sigma\right), \end{aligned} \tag{153}$$

and the variation ΔB of the total baryon number

$$B = i\int d^3x\, J^0_B \tag{154}$$

in the time interval Δt is related to the quantity $N_{\rm CS}$ and $n_{\rm CS}$, called the Chern-Simons numbers, in the same time interval

$$\Delta B = N_F(\Delta N_{\rm CS} - \Delta n_{\rm CS}), \tag{155}$$

where

$$\begin{aligned} N_{\rm CS} &= -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,{\rm Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right], \\ n_{\rm CS} &= -\frac{g_1^2}{16\pi^2}\int d^3x\,\epsilon^{ijk}\,\partial_i B_j B_k, \end{aligned} \tag{156}$$

where we have defined $A_i \equiv A^a_i\sigma^a/2$.

Now, each $U(1)_Y$ gauge transformation

$$B_i \to B_i + \frac{i}{g_1}(\partial_i U_Y)U_Y^{-1}, \tag{157}$$

with $U_Y(x) = e^{i\alpha_Y(x)}$ leaves $n_{\rm CS}$ unchanged. On the contrary, there exist $SU(2)_L$ gauge transformations

$$A_i \to U A_i U^{-1} + \frac{i}{g_2}(\partial_i U)U^{-1}, \tag{158}$$

which induce a nonvanishing variation of $N_{\rm CS}$

$$\delta N_{\rm CS} = \frac{1}{24\pi^2}\int d^3x\,{\rm Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}. \tag{159}$$

This is due to the topological properties of $SU(2)$. The most generic 2×2 unitary matrix with determinant equal to unity may be expressed as $a\mathbf{1} + ib_i\sigma^i$, with the condition $a^2 + |b|^2 = 1$. Therefore the topology of $SU(2)$ is the same as S^3, the surface of the hypersphere in four dimensions (three-sphere) and the gauge transformations are maps from the euclidean space onto $SU(2) \sim S^3$. To clarify this point further, we recall that classically, the ground state must correspond to *time-independent* field configuration with vanishing energy density. We have therefore $F^a_{\mu\nu} \equiv 0$, which means that the field $\mathbf{A}$ is a pure gauge, $\mathbf{A}_{\rm vac} = (i/g)(\nabla U)U^{-1}$ and we are working in the gauge $A_0 = 0$. Furthermore, we may restrict ourselves to the transformations U that have the same limit in all spatial directions. We may take this limit to be the identity in the group, $U \to \mathbf{1}$ as $|\vec{x}| \to \infty$. Under these circumstances, all the configurations $\mathbf{A}_{\rm vac}$ may be regarded as describing a ground state. We have seen that $SU(2)$ is isomorphic to the three-demensional sphere S^3. On the other hand, the whole three-dimensional space with all points at infinity identified is also topologically equivalent to S^3. Therefore the gauge transformation $U(x)$ associated with each vacuum is a mapping from S^3 onto S^3. According to the homotopy theory, such mappings fall into equivalence classes. Two mappings $\vec{x} \to U_1(\vec{x})$ and $\vec{x} \to U_2(\vec{x})$ belong to the same class if there exists a continuous transformation from $U_1(\vec{x})$ to $U_2(\vec{x})$. In the case at hand, the classes are labeled by positive or negative integer called the winding number.

We may consider some standard maps

$$\begin{aligned} U^{(0)}(x) &= 1, \\ U^{(1)}(x) &= \frac{x_0 + i\vec{x}\cdot\vec{\sigma}}{r}, \quad r = (x_0 + |\vec{x}|^2)^{1/2}, \\ &\vdots \\ U^{(n)}(x) &= \left[U^{(1)}\right]^n, \\ &\vdots \end{aligned} \tag{160}$$

It is easy to check that $\delta N_{\rm CS}$ vanishes for $U^{(0)}$ and any continous transformation of $U^{(0)}$

$$U(x) = U^{(0)}(x)\left(\mathbf{1} + i\epsilon^a(x)\sigma^a\right), \tag{161}$$

where $\epsilon^a(x) \to 0$ when $|\vec{x}| \to \infty$. On the other hand, $\delta N_{\rm CS}$ does not vanish if we consider the transformation $U^{(1)}$

$$\delta N_{\rm CS}\left(U^{(1)}\right) = 1. \tag{162}$$

The same result is obtained considering continuous deformations of $U^{(1)}$. It is also possible to show that – in general – the transformations $U^{(n)}$ and their continuous tranformations give rise to

$$\delta N_{\rm CS}\left(U^{(n)}\right) = n. \tag{163}$$

Therefore, the gauge transformations of $SU(2)$ may be divided in two categories, those which do not change the Chern-Simons number, and those which change the Chern-Simons number by n, the winding number.

Let us now consider the SM in the limit in which the mixing angle is zero, *i.e.* the theory is pure gauge $SU(2)_L$ theory coupled to the Higgs field Φ. If we choose the gauge $A_0 = 0$, we may go from the classical vacuum defined as

$$\mathcal{G}^{(0)}_{\rm vac} = \left\{A_i^{(0)} = 0, \Phi^{(0)} = (0, v), N_{\rm CS} = 0\right\}, \tag{164}$$

to an infinite number of other vacua

$$\mathcal{G}_{\rm vac}^{(n)} = \left\{ A_i^{(n)} = (i/g_2)(\nabla U^{(n)})(U^{(n)})^{-1}, \Phi^{(n)} = U^{(n)}\Phi^{(0)}, N_{\rm CS} = n \right\}, \tag{165}$$

which are classically degenerate and have different Chern-Simons number. Here we have denoted the vacuum expectation value (VEV) of the Higgs field by v.

If we now go back to eq. (155), we are able to understand the connection among the baryonic chiral anomaly, the topological structure of $SU(2)$ and the baryon number violation. If the system is able to perform a transition from the vacuum $\mathcal{G}_{\rm vac}^{(n)}$ to the closest one $\mathcal{G}_{\rm vac}^{(n\pm1)}$, the Chern-Simons number is changed by one unity and

$$\Delta B = \Delta L = N_F. \tag{166}$$

Each transition creates 9 left-handed quarks (3 color states for each generation) and 3 left-handed leptons (one per generation)

$$3\sum_{i=1}^{3} Q_L^i + \sum_{i=1}^{3} \ell_L^i \leftrightarrow 0. \tag{167}$$

6.3 The sphaleron

To quantify the probability of transition between two different vacua, it is important to understand the properties of the field configurations which interpolate the two vacua and "help" the transition. A fundamental result has been obtained in ref. [67], where it was found that there exist static configurations (therefore independent from t) which correspond to unstable solutions of the equations of motion. These solutions are called *sphalerons* (which in greek stands for "ready to fall") and correspond to saddle points of the energy functional and posses Chern-Simons number equal to 1/2. The situation is schematically depicted in Fig. 8

The sphaleron may be identified by considering the minimum energy path among all the paths that, in the configuration space, connect two vacua whose Chern-Simons number differs by one unit. Along this path, the sphaleron is the configuration of maximum energy and is localized is space, even though – contrary to the case of the soliton – is unstable.

In the limit of vanishing mixing angle, $\theta_W \to 0$, the sphaleron solution has been found explicitly by Klinkhamer and Manton [67] and has the following form

$$\begin{aligned} A_i dx^i &= \frac{i}{g_2} f(g_2 v r)\, U^\infty\, d(U^\infty)^{-1}, \\ \Phi &= \frac{iv}{\sqrt{2}} h(g_2 v r)\, U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \end{aligned} \tag{168}$$

where $U^\infty(x_1, x_2, x_3) = U^{(1)}(x_0 = 0, x_1, x_2, x_3)$. The energy functional

$$\begin{aligned} E &= \int d^3x \left[\frac{1}{4} F_{ij}^a F_{ij}^a + (\mathcal{D}_i\Phi)^\dagger(\mathcal{D}_i\Phi) + V(\Phi) \right], \\ V(\Phi) &= \lambda \left(\Phi^\dagger\Phi - \frac{1}{2}v^2 \right)^2, \end{aligned} \tag{169}$$

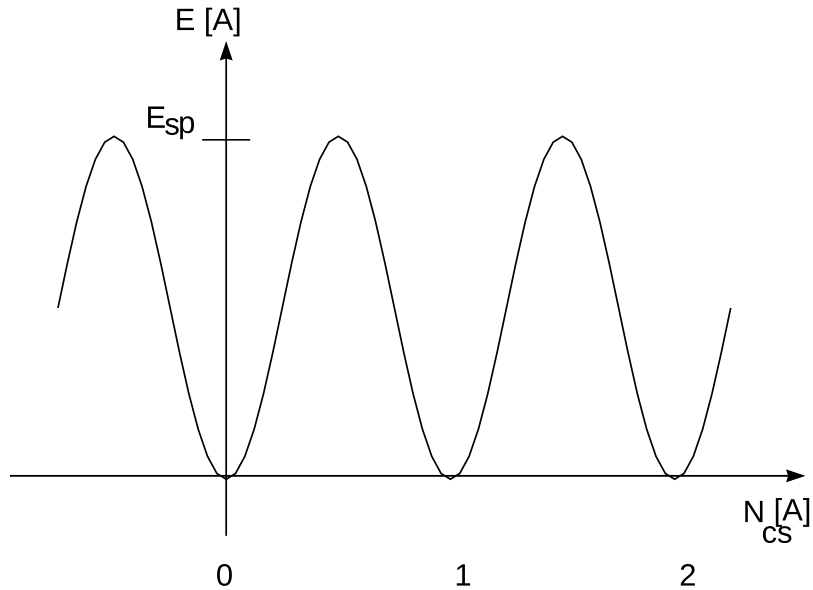


Figure 8: Schematical representation of the energy dependence of the gauge configurations as a function of the Chern-Simons number. Sphalerons correspond to the maxima of the curve.

may be reexpressed as

$$
\begin{aligned}
E &= \frac{4\pi v}{g_2}\int_0^\infty d\xi\,\left[4\left(\frac{df}{d\xi}\right)^2+\frac{8}{\xi^2}\left[f(1-f)\right]^2+\frac{1}{2}\xi^2\left(\frac{dh}{d\xi}\right)^2\right. \\
&+ \left.[h(1-f)]^2+\frac{\lambda}{g_2^2}\xi^2(h^2-1)^2\right], \\
\xi &= g_2 v r, \qquad (170)
\end{aligned}
$$

where $V(\phi)$ is the potential of the Higgs field.

The functions f and h are the ones which minimize the energy functional (170) with the boundary conditions

$$f(\xi)\to\begin{cases}\sim\xi^2, & \xi\to 0\\ 1, & \xi\to\infty\end{cases} \qquad (171)$$

and

$$h(\xi)\to\begin{cases}\sim\xi, & \xi\to 0\\ 1, & \xi\to\infty.\end{cases} \qquad (172)$$

The sphaleron is therefore the solution which interpolates between $\mathcal{G}_{\rm vac}^{(0)}$ (for $\xi\to 0$) and $\mathcal{G}_{\rm vac}^{(1)}$ (for $\xi\to\infty$). The energy and the typical dimensions of the sphaleron configuration are basically the result of the competition between the energy of the gauge configuration and the energy of the Higgs field. The latter introduces the weak scale into the problem. From the quantitative point of view, the potential energy of the Higgs field is less important and for the sphaleron configuration of dimension ℓ, we have

$$\begin{aligned} A_i &\sim \frac{1}{g_2 \ell}, \\ E(A_i) &\sim \frac{4\pi}{g_2^2 \ell}, \end{aligned} \tag{173}$$

while the energy of the Higgs field is

$$E(\phi) \sim 4\pi v^2 \ell. \tag{174}$$

Minimizing the sum $E(A_i) + E(\phi)$ we obtain that the typical dimension of the sphaleron is

$$\ell_{\rm sp} \sim \frac{1}{g_2 v} \sim 10^{-16} \ \ {\rm cm}, \tag{175}$$

and

$$E_{\rm sp} \sim \frac{8\pi v}{g_2} \sim 10 \ \ {\rm TeV}. \tag{176}$$

A more accurate result may be found by means of variational methods [67]

$$E_{\rm sp} = \frac{4\pi v}{g_2} B\left(\frac{\lambda}{g_2}\right), \tag{177}$$

where B is a function which depends very weakly on λ/g_2: $B(0) \simeq 1.52$ and $B(\infty) \simeq 2.72$. Including the mixing angle θ_W changes the energy of the sphaleron at most of 0.2%. The previous computation of the sphaleron energy was performed at zero temeprature. The sphaleron at finite temperature – but still in the broken phase – was computed in [16] where it was shown that its energy follows approximately the scaling law

$$E_{\rm sp}(T) = E_{\rm sp} \frac{\langle \phi(T) \rangle}{v}, \tag{178}$$

where $\langle \phi(T) \rangle$ is the VEV of the Higgs field at finite temperature in the broken phase. This energy may be writen as

$$E_{\rm sp}(T) = \frac{2 m_W(T)}{\alpha_W} B\left(\frac{\lambda}{g_2}\right), \tag{179}$$

where $m_W(T) = \frac{1}{2} g_2 \langle \phi(T) \rangle$.

The Chern-Simons number of the sphaleron may be explicitly computing by plugging (168) into (156) [67] and, as mentioned above, one obtains

$$N_{\rm CS}^{\rm sp} = \frac{1}{2}. \tag{180}$$

6.4 Baryon number violating transitions

The probability of baryon number nonconserving processes at zero temperature has been computed by 't Hooft [120] and, as we have already mentioned, is highly suppressed by a factor $\exp(-4\pi/\alpha_W) \sim 10^{-150}$. This factor may be interpreted as the probability of making a transition from one classical vacuum to the closest one by tunneling, by going through the barrier of ~ 10 TeV corresponding to the sphaleron. An easy way to evaluate this number is to remember that the field configurations that describe the transitions (sphalerons or instantons in the case of tunneling) are characterized by $A_i \sim 1/(g_2 v)$ and therefore their contribution to the generating function is

$$\int \mathcal{D}A\, e^{-S[A]} \sim e^{-1/\alpha_W} \ll 1. \tag{181}$$

On the other side one might think that baryon number violating transitions may be obtained in physical situations which involve a large number of fields. The contribution to the transition amplitude in processes which involve N fields may be estimated as

$$|A_{B+L}|^2 \sim \left(\frac{1}{\alpha_W}\right)^N e^{-4\pi/\alpha_W} = e^{-4\pi/\alpha_W - N\log\alpha_W}. \tag{182}$$

Therefore, if the number of fields involved is about $N \sim 1/\alpha_W$, the transition probability may become of order unity. The sphaleron may be produced by collective and coherent excitations containing $N \gtrsim 1/\alpha_W$ quanta with wavelength of the order of $\ell_{\rm sp} \sim 1/M_W$. At temperatures $T \gg M_W$, these modes essentially obey statistical mechanics and the transition probability may be computed via classical considerations. Note also that at temperatures $T \ll M_W$ it is no longer possible to deal with classical considerations because the Compton wavelength of the thermal excitations $\sim T^{-1}$ is much larger than the size of the sphaleron.

6.4.1 Baryon number violation below the electroweak phase transition

After (or during) the electroweak phase transition [106] by which the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ breaks down to $U(1)_{em}$, the calculation of the baryon number violation rate can be done by using the semiclassical approximations [85]. The vacuum expectation value of the Higgs field $\langle\phi(T)\rangle$ is nonvanishing and the sphaleron configuration may be explicitly written down.

We now want to estimate the transition probability between two different vacua havig Chern-Simons number which differ by one unity. One may use an useful analogy. Let us consider a pendulum of mass m and be θ the angle which determines the position of the pendulum with respect to the position at rest, $\theta = 0$. It is clear that the transformation

$$\theta \to \theta + 2\pi, \tag{183}$$

may be consider a sort of gauge transformation since the position θ and $\theta + 2\pi$ are indistinguishable. The periodic potential reads

$$V(\theta) = (mgh)(1 - \cos\theta), \tag{184}$$

wheer h is the pendulum length. The energy of the corresponding "sphaleron" – the saddle point solution to the equation of motion – is $V(\pi) = 2mgh$. According to the classical theory, for energies smaller than $V(\pi)$, only oscillations around $\theta = 0$ are possible. However, quantum theory predicts a nonvanishing probability of tunneling through the barrier separating $\theta = 0$ from $\theta = 2\pi$, *i.e.* a complete rotation of the pendulum.

Since the solution to the Schrodinger equation reads

$$\psi(\theta) = A\, e^{-\int_0^\theta d\theta' \sqrt{2mV(\theta')}} \tag{185}$$

the density probability $\mathcal{P}$ for penetration from $\theta = 0$ to $\theta = 2\pi$ is

$$\mathcal{P} \sim |\psi(2\pi)|^2 = |A|^2 e^{-2\int_0^{2\pi} d\theta \sqrt{2mV(\theta)}} \tag{186}$$

and the quantum tunneling is exponentially suppressed. Imagine now to raise up the temperature of the system, so that the pendulum coupled to the thermal bath becomes excited at higher and higher energies. As the temperature becomes of the order of $V(\pi)$, it becomes possible for the pendulum to reach the position $\theta = \pi$ and to roll down to $\theta = 2\pi$. The transition rate is therefore

$$\Gamma(T) \propto e^{-V(\pi)/T}, \tag{187}$$

and becomes unsuppressed as long as $T \gg V(\pi)$.

More formally, one has to remember that one of the fundamental objects in statistical thermodynamics is the partition function

$$Z = \mathrm{Tr}\, e^{-\beta\hat{H}}, \tag{188}$$

where $\hat{H}$ is the Hamiltonian operator and $\beta = T^{-1}$. For a scalar field, one may introduce the field eingenstates $|\phi(\vec{x}), t\rangle$ of the Heisenberg picture field operator $\hat{\phi}(\vec{x}, t)$

$$\hat{\phi}(\vec{x}, t)|\phi(\vec{x}), t\rangle = \phi(\vec{x})|\phi(\vec{x}), t\rangle. \tag{189}$$

Then the partition function may be written as "summation" over the eigenstates

$$Z = \sum_{\phi(\vec{x})} \langle\phi(\vec{x}), t = 0|e^{-\beta\hat{H}}|\phi(\vec{x}), t = 0\rangle. \tag{190}$$

We can now make the analogy with the zero temperature case where in the language of field theory

$$\begin{aligned} \langle\phi''(\vec{x}), t''|\phi'(\vec{x}), t'\rangle &= \langle\phi''(\vec{x}), t = 0|e^{-i\hat{H}(t''-t')}|\phi'(\vec{x}), t = 0\rangle \\ &\propto \int \mathcal{D}\phi\, \mathcal{D}\pi \exp\left[i\int_{t'}^{t''} dt \int d^3x \left(\pi\dot{\phi} - \mathcal{H}(\pi, \phi)\right)\right], \end{aligned} \tag{191}$$

where the path integral is over all the conjugate momenta of ϕ, π and over all the functions satisfying the boundary conditions $\phi = \phi''(\vec{x})$ at t'' and $\phi = \phi'(\vec{x})$ at t'. If, heuristically we introduce a variable

$$\tau \equiv it \tag{192}$$

and take the limit of integration

$$t' = 0 \quad \text{and} \quad t'' = -i\beta, \tag{193}$$

we obtain

$$\begin{aligned}&\langle\phi''(\vec{x}),t=0|e^{-\beta\hat{H}}|\phi'(\vec{x}),t=0\rangle\\ \propto\ &\int\mathcal{D}\phi\,\mathcal{D}\pi\,\exp\left[\int_0^\beta d\tau\int d^3x\left(i\pi\frac{\partial\phi}{\partial\tau}-\mathcal{H}(\pi,\phi)\right)\right],\end{aligned}\tag{194}$$

where now the new boundary conditions are given by

$$\phi(\beta,\vec{x})=\phi''(\vec{x})\quad\text{and}\quad\phi(0,\vec{x})=\phi'(\vec{x}).\tag{195}$$

By integrating out the conjugate momenta and identifying the boundary conditions, we may compute the partition function

$$Z\propto\int\mathcal{D}\phi\,\exp\left[\int_0^\beta d\tau\int d^3x\,\mathcal{L}\left(\phi,\overline{\partial}\phi\right)\right],\tag{196}$$

where

$$\overline{\partial}\phi\equiv\left(i\frac{\partial\phi}{\partial\tau},\nabla\phi\right)\tag{197}$$

and

$$\mathcal{L}(\phi,\overline{\partial}\phi)=-\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2-\frac{1}{2}\left(\nabla\phi\right)^2-V(\phi).\tag{198}$$

The theory at finite temperature may be therefore interpreted as a theory in 3+1 dimensions in the euclidean space with periodic boundary conditions on the time coordinate and period $\beta=1/T$.

In the limit of very high temperatures only the zero mode of the expansion $\phi=\beta^{-1}\sum_n e^{-i\omega_n\tau}\widetilde{\phi}(\omega_n,\vec{x})$, where $\omega_n=2\pi n/\beta$, is important and the action reduces to $-S_3/T$ where S_3 is euclidean three-dimensional action

$$S_3=\int d^3x\left[\frac{1}{2}(\nabla\phi)^2+V(\phi)\right].\tag{199}$$

The transition probability per unit time and unit volume at finite temperature between two different minima at ϕ_1 and ϕ_2 of a given potential $V(\phi)$ for a generic scalar field ϕ is therefore given at finite temperature by [85]

$$\frac{\Gamma}{V}\sim A(T)\,e^{-S_3/T},\tag{200}$$

where $A(T)$ is a prefactor which, on dimensional argument, is $\mathcal{O}(T^4)$ and the three-dimensional action must be computed for the field configuration (bounce solution) which interpolates between the two vacua

$$\begin{aligned}\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}&=V'(\phi),\\ \lim_{r\to\infty}\phi(r)&=\phi_1,\\ \left.\frac{d\phi}{dr}\right|_{r=0}&=0,\\ \phi(0)&=\phi_2,\\ r&=|\vec{x}|.\end{aligned}\tag{201}$$

This configuration is a bubble whose interior is characterized by the value of the scalar field ϕ_2 and the exterior by ϕ_1.

Exercise 3

Estimate the typical size R_c of the bounce solution in the limit of thick bubbles.

After this long disgression, we are ready to estimate the topological transition rate. Since the transition which violates the baryon number is is sustained by the sphaleron configuration, one gets $S_3 = E_{\rm sp}(T)$. The prefactor was computed in [21] as

$$\Gamma_{\rm sp} \sim 2.8 \times 10^5\, T^4 \left(\frac{\alpha_W}{4\pi}\right)^4 \kappa \left[\frac{E_{\rm sp}(T)}{B}\right]^7 e^{-E_{\rm sp}(T)/T}, \tag{202}$$

where B has been defined in (170) and κ is the functional determinant associated to the fluctuations about the sphaleron. It has been estimated to be in the range $10^{-4} \lesssim \kappa \lesssim 10^{-1}$ [37].

6.4.2 Baryon number violation above the electroweak phase transition

At temperatures above the electroweak phase transition, the vacuum expectation value of the Higgs field is zero, $\langle\phi(T)\rangle = 0$, the Higgs field decouples and the sphaleron configuration ceases to exist. The relevant configuration, at this point, are of the form

$$\begin{aligned} \Phi &= 0, \\ A_i dx^i &= \frac{i}{g_2}\, f\, U^\infty\, d(U^\infty)^{-1}. \end{aligned} \tag{203}$$

Let us estimate the rate $\Gamma_{\rm sp}$ on dimensional grounds. As we mentioned, at high temperature T the Higgs field decouples from the dynamics and it suffices to consider a pure $SU(2)$ gauge theory. Topological transitions take place through the creation of non-perturbative, nearly static, magnetic field configurations that generate a change in the Chern-Simons number $\Delta N_{\rm CS}$ with a corresponding baryon number generation $\Delta B = N_f \Delta N_{\rm CS}$.

If the field configuration responsible for the transition has a typical scale ℓ, a change $\Delta N_{\rm CS} \simeq 1$ requires

$$\Delta N_{\rm CS} \sim g_2^2\, \ell^3\, \partial A_i A_i \sim g_2^2 \ell^3\, \frac{A_i}{\ell} A_i \sim 1 \Rightarrow A_i \sim \frac{1}{g_2 \ell}. \tag{204}$$

This means that the typical energy of the configuration is

$$E_{\rm sp} \sim \ell^3\, (\partial A_i)^2 \sim \frac{1}{g_2^2 \ell}. \tag{205}$$

To evade the Boltzmann suppression factor this energy should not be larger than the temperature T, which requires

$$\ell \gtrsim \frac{1}{g_2^2 T}. \tag{206}$$

Such a length scale corresponds to the one of the dynamically generated magnetic mass of order $g_2^2 T$ which behaves as a cut off for the maximum coherence length of the system. The rate of one unsuppressed transition per volume ℓ^3 and time $t \sim \ell$ is therefore

$$\Gamma_{\rm sp} \sim \frac{1}{\ell^3 t} \sim (\alpha_W\, T)^4. \tag{207}$$

This simple scaling argument has been recently criticized in refs. [5, 6] where it has been argued that damping effects in the plasma suppress the rate by an extra power of α_W to give $\Gamma_{\rm sp} \sim \alpha_W^5 T^4$. Indeed, since the transition rate involves physics at soft energies $g_2^2 T$ that are small compared to the typical hard energies $\sim T$ of the thermal excitations in the plasma, the simplest way of analyzing the problem is to consider an effective theory for the soft modes, where the hard modes have been integrated out and to keep the dominant contributions, the so-called hard thermal loops [14]. It is the resulting typical frequency ω_c of a gauge field configuration immersed in the plasma and spatial with extent $(g^2 T)^{-1}$ that determines the change of baryon number per unit time and unit volume. This frequency ω_c has been estimated to be $\sim g_2^4 T$ when taking into account the damping effects of the hard modes [5, 6]. This gives $\Gamma_{\rm sp} \sim \frac{\omega_c}{\ell^3} \sim \alpha_W^5 T^4$. The effective dynamics of soft nonabelian gauge fields at finite temperature has been recently addressed also in [13], where it was found that $\Gamma_{\rm sp} \sim \alpha_W^5 T^4 \ln(1/\alpha_W)$. Lattice simulations with hard-thermal loops included have been performed [98] and seem to indicate the $\Gamma_{\rm sp} \sim 30\alpha_W^5 T^4$, which is not far from $\alpha_W^4 T^4$. In order to see whether these predictions are reliable, one should write down an effective classical hamiltonian for the soft modes of the gauge configurations after having integrated out also the soft loops between magnetic fields. These soft loops result to be crucial since they not only renormalize the effective α_W coupling to non perturbative values, but also falsify the naive dimensional arguments about the typical time scale of the sphaleron-like fluctuations. From now on, we will parametrize the sphaleron rate as

$$\Gamma_{\rm sp} = \kappa (\alpha_W\, T)^4. \tag{208}$$

6.5 The wash-out of $B + L$

Let us suppose – for sake of simplicity – that all the charges which are conserved by the interactions of the particles in the plasma (Q, L_i, $B - L$, $\ell_i = B/3 - L_i$, $\cdots$) are zero. If we introduce a chemical potential for the charge $B + L$, μ_{B+L}, the free energy density of the system (femions) is given by

$$F = T \int \frac{d^3k}{(2\pi)^3} \left[\log\left(1 + e^{-(E_k - \mu_{B+L})/T}\right) + (\mu_{B+L} \to -\mu_{B+L})\right]. \tag{209}$$

The charge density of $B + L$ may be expressed in terms of the chemical potential by

$$n_{B+L} \sim \mu_{B+L} T^2 \tag{210}$$

and – therefore – we may relate the free energy with n_{B+L}

$$F \sim \mu_{B+L}^2 T^2 + \mathcal{O}(T^4) \sim \frac{n_{B+L}^2}{T^2} + \mathcal{O}(T^4). \tag{211}$$

The free energy increases quadratically with the fermion number density and the transitions which increase n_{B+L} are energetically disfavoured with respect to the ones that decrease the fermion number. If these transitions are active for a long enough period of time, the system relaxes to the state of minimum energy, *i.e.* $n_{B+L} = 0$: *any initial asymmetry in $B + L$ relaxes to zero.*

To address this issue more quantitatively, one has to consider the ratio between the transitions with $\delta N_{\rm CS} = +1$ and the ones with $\delta N_{\rm CS} = -1$

$$\frac{\Gamma_+}{\Gamma_-} = e^{-\Delta f/T}, \tag{212}$$

wheer Δf is the free energy difference between the two vacua. If we define $\Gamma_{\rm sp}$ to be the average between Γ_+ and Γ_-, we may compute the rate at which the baryon number is washed out [10]

$$\frac{dn_{B+L}}{dt} = \Gamma_+ - \Gamma_- \simeq -\frac{13}{2}\, N_F\, \frac{\Gamma_{\rm sp}}{T^3}\, n_{B+L}. \tag{213}$$

Equation (213) is crucial to discuss the fate of the baryon asymmetry generated at the GUT scale and is called *Master* equation.

Let us now consider temperatures much above the electroweak phase transition, $T \gg M_W$. Baryon number violation processes are active at very high temperatures if the rate 207) is smaller than the expansion of the Universe

$$\frac{\Gamma_{\rm sp}}{T^3} \gtrsim H \Rightarrow T \lesssim \alpha_W^4\, \frac{M_{\rm P}}{g_*^{1/2}} \sim 10^{12}\ {\rm GeV}. \tag{214}$$

If so, any preexisting asymmetry in $B + L$ is erased exponentially with a typical time scale $\tau \sim 2N_F T^3/13\Gamma_{\rm sp}$.

Let us now consider temperatures $T \sim M_W$ when the electroweak phase transition is taking place and the Higgs VEV $\langle\phi(T)\rangle$ is not zero. Baryon number violation processes are out-of-equilibrium if, again, the rate (202) is smaller than the expansion rate of the Universe. This translates into the bound on $E_{\rm sp}(T)$ [10]

$$\frac{E_{\rm sp}(T_c)}{T_c} \gtrsim 45, \tag{215}$$

wheer we have indicated by T_c the critical temperature at which the electroweak phase transition is taking place. Using the relation (179) this bound may be translated into a bound on $\langle\phi(T_c)\rangle$

$$\frac{\langle\phi(T_c)\rangle}{T_c} \gtrsim 1. \tag{216}$$

Any generation of the baryon asymmetry at the electroweak phase transition requires – therefore – a strong enough phase transition, that is able to produce a VEV for the Higgs field larger than the critical temperature. We will come back to this point later on.

6.5.1 A crucial point

In all the considerations leading to Eq. (213) we have been assuming that all the charges which are conserved by the interactions of the particles in the plasma (Q, $B - L$, L_i,

$\ell_i = B - L_i/3, \cdots$) are vanishing. Suppose now that these charges – let us denote them generically by Q_i – are not zero. Define by $(B+L)_{\rm EQ}$ the value of the number density associated to the $B+L$ charge when the sphaleron transitions are in *equilibrium* in the plasma (ideally, when the sphaleron rate $\Gamma_{\rm sp} \to \infty$). In such a case, it is possible to show (see Exercise 4) that $(B+L)_{\rm EQ}$ is not vanishing in the plasma [62]

$$(B+L)_{\rm EQ} = \sum_i c_i\, Q_i, \tag{217}$$

where the numerical coefficients c_i depend upon which interactions are in equilibrium in the plasma and the particle content of the theory.

Eq. (217) tells us that anomalous baryon number violating processes do not wash out completely the combination $B+L$ if at least one of the charges which are conserved by the interactions of the plasma, *e.g.* $(B-L)$ is nonvanishing. Eq. (213) changes accordingly (see Exercise 4):

$$\frac{dn_{B+L}}{dt} \propto -\frac{\Gamma_{\rm sp}}{T}\frac{\partial F}{\partial(B+L)} \propto -\frac{\Gamma_{\rm sp}}{T}\left[n_{(B+L)} - n^{\rm EQ}_{(B+L)}\right]. \tag{218}$$

The interpretation of this equation is straightforward. It is a Boltzmann equation in the sense that, in the limit $\Gamma_{\rm sp} \to \infty$, the solution is $B+L = (B+L)_{\rm EQ}$. If the conserved charges are zero, then any $B+L$ is washed-out, see Eq. (217). However, if $(B+L)_{\rm EQ}$ is not zero, then sphalerons transitions will act on the system until the $(B+L)$ charge has been reduced to its equilibrium value. The latter is not necessarily zero if some other conserved charge, like $B-L$, is not zero. In other words, sphaleron transitions push the system towards the state of minimum free energy, which is characterized by a nonvanishing $B+L$ if other conserved charges are non zero. This is a crucial point to keep in mind when we will talk about electroweak baryogenesis.

Exercise 4

a) Consider the two Higgs doublet model in the broken phase. The Higgs doublets are defined as $H_1 = (H_1^0, H^-)^T$ and $H_2 = (H^+, H_2^0)^T$ and couple to the down-type and up-type quarks, respectively. By considering all the processes in thermal equilibrium (but the ones mediated by light quark Yukawa interactions, Cabibbo suppressed gauge interactions and sphalerons transitions), identify the charges which are conserved by the interactions. *Hint*: one of them is $B+L$; *b)* assuming that also the sphaleron transitions are in equilibrium, compute the relation between the corresponding equilibrium value of the $B+L$ charge (call it $(B+L)_{\rm EQ}$), and the other conserved charges. *c)* Compute the free energy of the system and show that it scales like $[(B+L)-(B+L)_{\rm EQ}]^2$.

6.6 Baryon number violation within the SM and GUT baryogenesis

At this point, we are ready to discuss the implications of the baryon number violation in the early Universe for the baryogenesis scenarios discussed so far. The basic lesson we have learned in the previous subsections is that any asymmetry $B+L$ is rapidly erased by sphaleron transitions as soon as the temperatures drops down $\sim 10^{12}$GeV. Now, we can always write the baryon number B as

$$B = \frac{B+L}{2} + \frac{B-L}{2}. \tag{219}$$

This equation seems trivial, but is dense of physical significance! Sphaleron transitions only erase the combination $B+L$, but leave untouched the orthogonal combination $B-L$. This means that the only chance for a GUT baryogenesis scenario to work is to produce at high scale an asymmetry in $B-L$. In section 4 – however – we have learned that there is no possibility of generating such an asymmetry in the framework of $SU(5)$. This is because the fermionic content of the theory is the one of the SM and there is no violation of $B-L$. Sphaleron transitions are therefore the killers of any GUT baryogenesis model based on the supersymmetric version of $SU(5)$ with R parity conserved (The non-supersymmetric version is already ruled out by experiments on the proton decay lifetime). This is a striking result.

6.6.1 Baryogenesis via leptogenesis

The fact that the combination $B-L$ is left unchanged by sphaleron transitions opens up the possibility of generating the baryon asymmetry from a lepton asymmetry. This was suggested by Fukugita and Yanagida [47]. The basic idea is that, if an asymmetry in the lepton number is produced, sphaleron transition will reprocess it and convert (a fraction of) it into baryon number. This is because $B+L$ must be vanishing all the times and therefore the final baryon asymmetry results to be $B \simeq -L$. The primordial lepton asymmetry is generated by the out-of-equilibrium decay of heavy right-handed Majorana neutrinos N_L^c. Once the lepton number is produced, the processes in thermal equilibrium distribute the charges in such a way that in the high temperature phase of the standard model the asymmetries of baryon number B and of $B-L$ are proportional in thermal equilibrium [62] (see Eq. (217)

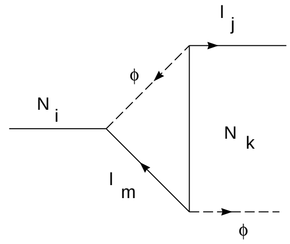
$$B = \left(\frac{8N_F + 4N_H}{22N_F + 13N_H}\right)(B-L), \tag{220}$$

Where N_H is the number of Higgs doublets. As we have already stressed, in the standard model, as well as its unified extension based on the group $SU(5)$, $B-L$ is conserved. Hence, no asymmetry in $B-L$ can be generated, and B vanishes. However, a nonvanishing $B-L$ asymmetry may be naturally obtained adding right-handed Majorana neutrinos to the standard model. This extension of the standard model can be embedded into GUTs with gauge groups containing $SO(10)$. Heavy right-handed Majorana neutrinos can also explain the smallness of the light neutrino masses via the see-saw mechanism [50].

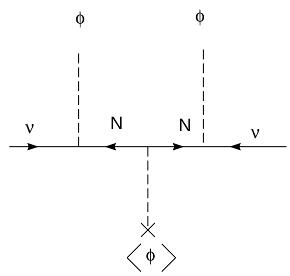
The basic piece of the Lagrangian that we need to understand leptogenesis is the coupling between the right-handed neutrino, the Higgs doublet Φ and the lepton doublet ℓ_L

$$\mathcal{L} = \overline{\ell_L}\,\Phi\,h_\nu\,N_L^c + \frac{1}{2}\overline{N_L^c}\,M\,N_L^c + \text{h.c.} \tag{221}$$

The vacuum expectation value of the Higgs field $\langle\Phi\rangle$ generates Dirac masses m_D for neutrinos $m_D = h_\nu\langle\Phi\rangle$, which are assumed to be much smaller than the Majorana masses



(a)



(b)

Figure 9: (a) One-loop interference giving rise to the lepton asymmetry; (b) Diagram giving rise to the 5-dimensional operator of Eq. (216).

M. When the Majorana right-handed neutrinos decay into leptons and Higgs scalars, they violate the lepton number (right-handed neutrino fermionic lines do not have any preferred arrow)

$$\begin{aligned} N_L^c &\rightarrow \overline{\Phi} + \ell, \\ N_L^c &\rightarrow \Phi + \overline{\ell}. \end{aligned} \tag{222}$$

The interference between the tree-level and the one-loop amplitudes, see Fig. 9(a), yields a CP asymmetry equal to

$$\frac{1}{8\pi v^2 \left(m_D^\dagger m_D\right)_{ii}} \sum_j \mathrm{Im}\left[\left(m_D^\dagger m_D\right)_{ij}^2\right] f\left(\frac{M_j^2}{M_i^2}\right), \tag{223}$$

where

$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right] \tag{224}$$

and the index i is summed over all the three species of right-handed neutrino. The final baryon asymmetry has been computed by several authors [91, 81, 30, 17] and it has been shown to be of the order of

$$B \simeq (0.6 - 1) \times 10^{-10}. \tag{225}$$

However, one has to avoid a large lepton number violation at intermediate temperatures which may potentially dissipate away the baryon number in combination with the sphaleron transitions. Indeed, the diagram of Fig. 9(b), induced by the exchange of a heavy right-handed neutrino, gives rise to a $\Delta L = 2$ interaction of the form

$$\frac{m_\nu}{\langle\Phi\rangle^2}\ell_L\ell_L\Phi\Phi + \text{h.c.}, \tag{226}$$

where m_ν is the mass of the light left-handed neutrino. The rate of lepton number violation induced by thi interaction is therefore $\Gamma_L \sim (m_\nu^2/\langle\Phi\rangle^4)T^3$. The requirement of harmless letpon number violation, $\Gamma_L \lesssim H$ imposes an interesting bound on the neutrino mass

$$m_\nu \lesssim 4\,\text{eV}\left(\frac{T_X}{10^{10}\,\text{GeV}}\right)^{-1/2}, \tag{227}$$

where $T_X \equiv \text{Min}\left\{T_{B-L}, 10^{12}\text{ GeV}\right\}$ and T_{B-L} is the temperature at which the $B-L$ number production takes place and $\sim 10^{12}$ GeV is the temperature at which sphaleron transitions enter in equilibrium. One can also reverse the argument and study leptogenesis assuming a similar pattern of mixings and masses for leptons and quarks, as suggested by $SO(10)$ unification [17]. This implies that $B-L$ is broken at the unification scale $\sim 10^{16}$ GeV, if $m_{\nu_\mu} \sim 3\times 10^{-3}$ eV as preferred by the MSW explanation of the solar neutrino deficit [125, 97].

7 Electroweak baryogenesis

So far, we have been assuming that the departure from thermal equilibrium, necessary to generate any baryon asymmetry, is attained by late decays of heavy particles. In this section, we will focus on a different mechanism, namely the departure from equilibrium during first order phase transitions.

A first order phase transition is defined to occur if some thermodynamic quantities change discontinuously. This happens because there exist two separate thermodynamic states that are in thermal equilibrium at the time of the phase transition. The thermodynamic quantity that undergoes such a discontinuous change is generically called the order parameter ϕ. Whether a phase transition is of the first order or not depends upon the parameters of the theory and it may happen that, changing those parameters, the order parameter becomes continuous at the time of the transition. In this case, the latter is said to be of the second order at the point at which the transition becomes continuous and a continuous crossover at the other points for which all physical quantities undergo no changes. In general, we are interested in systems for which the high temperature ground state of the theory is at $\phi = 0$ and the low temperature phase is at $\phi \neq 0$ [73, 106].

For a first order phase transition, the extremum at $\phi = 0$ becomes separated from a second local minimum of the potential by an energy barrier. At the critical temperature T_c both phases are equally favoured energetically and at later times the minimum at $\phi \neq 0$ becomes the global minimum of the theory. The phase transition proceeds by nucleation of bubbles. Initially, the bubbles are not large enough for their volume energy to overcome the competing surface tension, they shrink and disappear. However, at the nucleation

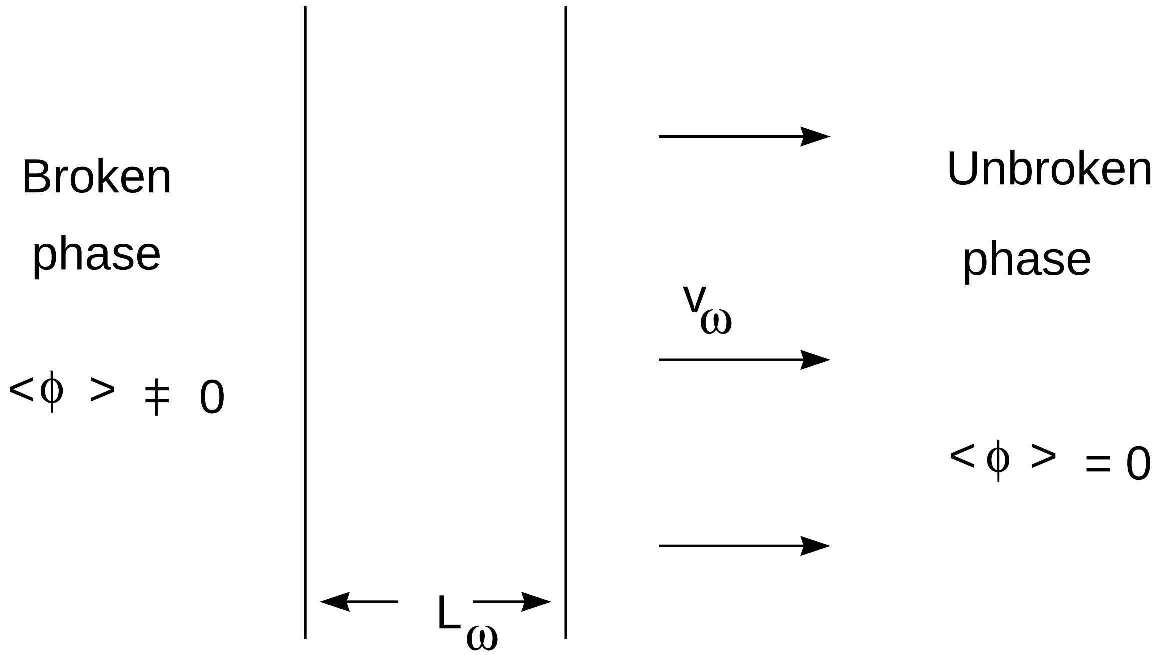


Figure 10: Schematic picture of the propagating bubble separating the broken from the unbroken phase during the electroweak phase transition.

temperature, critical bubbles form, *i.e.* bubbles which are just large enough to be nucleated and to grow. As the bubble walls separating the broken from the unbroken phase pass each point in space, the order parameter changes rapidly, leading to a significant departure from thermal equilibrium.

The critical bubbles of the broken (Higgs) phase have a typical profile

$$\phi(r) = \frac{\langle \phi(T_c) \rangle}{2} \left[1 + \tanh \left(\frac{r}{L_\omega} \right) \right] , \tag{228}$$

where r is th spatial coordinate, L_ω is the bubble wall width and $\langle \phi(T_c) \rangle$ is the VEV of the Higgs field inside the bubble.

Bubbles expand with velocity v_ω until the fill the Universe; local departure from thermal equilibrium takes place in the vicinity of the expanding bubble walls, see Fig. 10.

The fundamental idea of electroweak baryogenesis is to produce asymmetries in some local charges which are (approximately) conserved by the interactions inside the bubble walls, where local departure from thermal equilibrium is attained. These local charges will then diffuse into the unbroken phase where baryon number violation is active thanks to the unsuppressed sphaleron transitions. The latter convert the asymmetries into baryon asymmetry, because the state of minimum free energy is attained for nonvanishing baryon number, see eq. (218). Finally, the baryon number flows into the broken phase where it remains as a remnant of the electroweak phase transition if the sphaleron transitions are suppressed in the broken phase. The recipe for electroweak baryogenesis is therefore the following:

– Look for those charges which are approximately conserved in the symmetric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, so that the generation of a non-zero baryon charge is energetically favoured.

– Compute the CP violating currents of the plasma locally induced by the passage of the bubble wall.

– Write and solve a set of coupled differential diffusion equations for the local particle densities, including the CP violating source terms derived from the computation of the current at the previous step and the particle number changing reactions. The solution to these equations gives a net baryon number which is produced in the symmetric phase and then transmitted into the interior of the bubbles of the broken phase, where it is not wiped out if the first transition is strong enough.

7.1 Electoweak baryogenesis in the SM

Since C and CP are known to be violated by the electroweak interactions, it is possible – in principle –to satisfy all Sakharov's conditions within the SM if the electroweak phase transition leading to the breaking of $SU(2)_L \otimes U(1)_Y$ is of the first order [62]. There are very good reviews on electroweak baryogenesis and the reader is referred to them for more details [29, 113, 122, 38].

The asymmetry flowing inside the bubbles of the broken phase will survive if sphaleron transitions are frozen out and baryon number violation is inefficient. As we have learned in the previous section, baryon number violation is out-of-equilibrium inside the bubble wall only of $\frac{\langle\phi(T_c)\rangle}{T_c} \gtrsim 1$, *i.e.* if the electoweak phase transition is strong first order. Let us now understand as this condition translates into a *upper* bound on the Higgs mass m_h.

In general, given an order parameter ϕ and a set of particles i with masses $m_i(\phi)$ in the ϕ background, plasma masses $\pi_i(T)$ and degrees of freedom n_i, the effective one-loop improved potential at finite temperature is given by [106]

$$\Delta V^{\rm bos}(\phi,T) = \sum_i n_i \left\{ \frac{m_i^2(\phi)}{24}T^2 - \frac{T}{12\pi}\left[m_i^2(\phi)+\Pi_i(T)\right]^{3/2} - \frac{m_i^4(\phi)}{64\pi^2}\log\frac{m_i^2(\phi)}{A_B T^2}\right\} \quad (229)$$

if the particles are bosons and

$$\Delta V^{\rm fer}(\phi,T) = \sum_i n_i \left\{ \frac{m_i^2(\phi)}{48}T^2 + \frac{m_i^4(\phi)}{64\pi^2}\log\frac{m_i^2(\phi)}{A_F T^2}\right\} \quad (230)$$

if they are fermions. Here $A_B = 16\,A_F = 16\pi^2\exp(3/2-2\gamma_E)$, $\gamma_E \simeq 0.5722$.

One can therefore write the total one-loop effective potential of the SM Higgs field at finite temperature as as [106]

$$V(\phi,T) = D(T^2-T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4, \quad (231)$$

where

$$D = \frac{2M_W^2+M_Z^2+2m_t^2}{8v^2},$$

$$
\begin{aligned}
E &= \frac{2M_W^3+M_Z^3}{4\pi v^3},\\
T_0 &= \frac{m_h^2-8Bv^2}{4D},\\
B &= \frac{3}{64\pi^2 v^4}(2M_W^4+M_Z^4-4m_t^4),\\
\lambda(T) &= \lambda-\frac{3}{16\pi^2 v^4}\left(2M_W^4\,{\rm Log}\frac{M_W^2}{A_BT^2}+M_Z^4\,{\rm Log}\frac{M_Z^2}{A_BT^2}-4m_t^4\,{\rm Log}\frac{M_t^2}{A_FT^2}\right),
\end{aligned}
\tag{232}
$$

where m_t is the mass of the top-quark.

It is now easy to see that, when the minimum $\phi=0$ becomes metastable, *i.e* at the temperature T_c when $V(0,T_c)=V(\phi(T_c),T_c)$, one has

$$
\frac{\phi(T_c)}{T_c}=\frac{2ET_c}{\lambda(T_c)}\simeq\frac{4Ev^2}{m_h^2},
\tag{233}
$$

where we have used the fact that $m_h^2=2\lambda v^2$. The condition (216) is therefore satisfied only if

$$
m_h \lesssim \sqrt{\frac{4E}{1.3}}\sim 42\,{\rm GeV}.
\tag{234}
$$

On the other hand, the current lower bound on m_h comes from combining the results of DELPHI, L3 and OPAL experiments and is $m_h>89.3$ GeV [11]. A simple one-loop computation shows, therefore, that the electroweak phase transition is too weakly first order to assure the preservation of the generated baryon asymmetry at the electroweak phase transition in the SM. More complete perturbative and non-perturbative analyses [113] have shown that the electroweak phase transition is first order if the mass of the Higgs m_h is smaller than about 80 GeV and for larger masses becomes a smooth crossover. Let us now briefly analyzed the issue of CP violation within the SM. Because of CP violation in the kaon system, it is of great interest to see whether enough CP violation is present in the SM to generate the baryon asymmetry at the observed level.

A very rough (and optimistic) estimate of the amount of CP violation necessary to generate $B\simeq 10^{-10}$ can be obtained as follows. Since the baryon number violation rate in the symmetric phase is proportional to $\alpha_W^4\simeq 10^{-6}$, if we indicate by δ_{CP} the suppression factor due to CP violation, we get

$$
B\simeq\frac{\alpha_W^4\,T^3}{s}\,\delta_{CP}\simeq 10^{-8}\,\delta_{CP}.
\tag{235}
$$

Even neglecting all the suppression factors coming from the dynamics of the electroweak phase transition, we discover that

$$
\delta_{CP}\gtrsim 10^{-3}.
\tag{236}
$$

A naive estimate suggests that, since CP violation vanishes in the SM if any two quarks of the same charge have the same mass, the measure of CP violation should be the Jarlskog invariant

$$
\begin{aligned}
\frac{A_{CP}}{J} &= \left(M_t^2-M_c^2\right)\left(M_c^2-M_u^2\right)\left(M_u^2-M_t^2\right)\\
&\quad \left(M_b^2-M_s^2\right)\left(M_s^2-M_d^2\right)\left(M_d^2-M_b^2\right),
\end{aligned}
\tag{237}
$$

where J is twice the area of the unitarity triangle. The quantity A_{CP} has dimension twelve. In the limit of high temperature, T much larger than the quark masses M_q, the only mass scale in the problem is the temperature itself. Therefore, the dimensionless quantity δ_{CP} is

$$\delta_{CP} \simeq \frac{A_{CP}}{T_c^{12}} \simeq 10^{-20}, \tag{238}$$

far too small for the SM to explain the observed baryon asymmetry.

This admittedly too naive reasoning has been questioned by Farrar and Shaposhnikov [45] who have pointed out that for quarks having momentum $p \sim T \gg M_q$, the above estimate is certainly correct since light quarks are effectively degenerate in mass and the GIM suppression is operative; on the other side, this is no longer true when quarks have a momentum $p \sim M_q$. Since the mass jump through the bubble wall is just M_q, quarks coming from the symmetric phase and with momentum $p < M_q$ are reflected off from the wall, while the ones with momentum $p > M_q$ are partially reflected and partially trasmitted. In the reflection processes quarks and antiquarks acquire different probabilities of penetrating the bubble wall. In such a way, it might be possible to produce a net baryon number flux from outside to inside the bubble wall. For instance, considering momenta between M_d and M_s, then all the strange quarks might be reflected off, while down quarks have a nonvanishing probability of being transmitted. However, this effect is largely suppressed by the fact that fermions, when they propagate in the plasma, acquire a damping rate $\gamma \sim 0.1\,T \gg M_s$ and the quark energy and momenta cannot be defined exactly, but have a spread of the order of $\gamma \gg (M_s - M_d)$. In other words, the lifetime of the quantum packet is much shorter than the typical reflection time from the bubble wall ($\sim 1/M_s$): CP violation, which is based on coherence and needs at least a time $\sim 1/M_s$ to be built up, cannot be efficient [49, 59]. Therefore, the common wisdom is that electroweak baryogenesis is not possible within the SM.

7.2 Electoweak baryogenesis in the MSSM

The most promising and well-motivated framework for electroweak baryogenesis beyond the SM seems to be supersymmetry (SUSY) [56, 4]. Let us remind the reader only a few notions about the MSSM that will turn out to be useful in the following.

Let us consider the MSSM superpotential

$$W = \mu \hat{H}_1 \hat{H}_2 + h^u \hat{H}_2 \hat{Q} \hat{u}^c + h^d \hat{H}_1 \hat{Q} \hat{d}^c + h^e \hat{H}_1 \hat{L} \hat{e}^c, \tag{239}$$

where we have omitted the generation indices. The Higg sector contains the two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^0 \\ H^- \end{pmatrix} \quad \text{and} \quad H_2 = \begin{pmatrix} H^+ \\ H_1^0 \end{pmatrix}. \tag{240}$$

The lepton Yukawa matrix h^e can be always taken real and diagonal while h^u and h^d contain the KM phase.

What is relevant for baryogenesis is to identify possible new sources of CP violation. They emerge from the operators which break softly supersymmetry

i) Trilinear couplings:

$$\Gamma^u H_2 \widetilde{Q}\widetilde{u}^c + \Gamma^d H_1 \widetilde{Q}\widetilde{d}^c + \Gamma^e H_1 \widetilde{L}\widetilde{e}^c + \text{h.c.}, \tag{241}$$

where we have defined

$$\Gamma^{(u,d,e)} \equiv m_{3/2}\, A^{(u,d,e)} \cdot h^{(u,d,e)}. \tag{242}$$

Generally, in supergravity models the matrices $A^{(u,d,e)}$ are assumed to be proportional to the identiy matrix

$$A^{(u,d,e)}(M_{\rm GUT}) = A \cdot \mathbf{1}, \tag{243}$$

where the A parameter can be complex.

ii) bilinear couplings:

$$\mu B H_1 H_2 + \text{h.c.} \tag{244}$$

iii) Majorana gaugino masses:

$$\frac{1}{2}\left(M_1\lambda_1\lambda_1 + M_2\lambda_2\lambda_2 + M_3\lambda_3\lambda_3\right) + \text{h.c.} \tag{245}$$

At the GUT scale it is usually assumed that

$$M_1 = M_2 = M_3 = M. \tag{246}$$

iv) Scalar soft masses:

$$m^2_{ab}\widetilde{z}^*_a \widetilde{z} + \text{h.c.} \tag{247}$$

The new contributions to explicit violation of CP are given in the phases of the complex parameters A, B, M_i $(i = 1,2,3)$ and by the parameter μ in the superpotential (239). Two phases may be removed by redifining the phase of the superfield $\hat{H}_2$ in such a way that the phase of μ is opposite to that of B. The product μB in (244) is therefore real. It is also possible to remove the phase of the gaugino mass M by an R symmetry transformation. The latter leaves all the other supersymmeric couplings invariant and only modifies the trilinear ones, which get multiplied by $\exp(-\phi_M)$ where ϕ_M is the phase of M.

The phases which are left are therefore

$$\phi_A = \arg(AM) \;\; \text{and} \;\; \phi_\mu = -\arg(B). \tag{248}$$

The two new phases ϕ_A and ϕ_μ will be crucial for the generation of the baryon asymmetry.

Electroweak baryogenesis in the framework of the Minimal Supersymmetric Standard Model (MSSM) has attracted much attention in the past years, with particular emphasis on the strength of the phase transition [51, 100, 42, 15] and the mechanism of baryon number generation [102, 19, 108, 109, 110, 111, 28].

Recent analytical [18, 33, 43, 12, 87, 88, 44, 20, 89] and lattice computations [77, 27, 78, 79] have revealed that the phase transition can be sufficiently strongly first order if the ratio of the vacuum expectation values of the two neutral Higgses $\tan\beta$ is smaller than ~ 4. Moreover, taking into account all the experimental bounds as well as those coming from the requirement of avoiding dangerous color breaking minima, the lightest Higgs boson should be lighter than about 105 GeV, while the right-handed stop mass might be close to the present experimental bound and should be smaller than, or of the order of, the top quark mass [20].

Moreover, as we have seen, the MSSM contains additional sources of CP-violation besides the CKM matrix phase. These new phases are essential for the generation of the baryon number since large CP violating sources may be locally induced by the passage of the bubble wall separating the broken from the unbroken phase during the electroweak phase transition. Baryogenesis is fuelled when transport properties allow the CP violating charges to efficiently diffuse in front of the advancing bubble wall where anomalous electroweak baryon violating processes are not suppressed. The new phases appear in the soft supersymmetry breaking parameters associated to the stop mixing angle and to the gaugino and neutralino mass matrices; large values of the stop mixing angle are, however, strongly restricted in order to preserve a sufficiently strong first order electroweak phase transition. Therefore, an acceptable baryon asymmetry from the stop sector may only be generated through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter, and their associated CP violating phases [19]. As a result, the contribution to the final baryon asymmetry from the stop sector turns out to be negligible. On the other hand, charginos and neutralinos may be responsible for the observed baryon asymmetry if the phase of the parameter μ is large enough [19, 28]. Yet, this is true within the MSSM. If the strength of the electroweak phase transition is enhanced by the presence of some new degrees of freedom beyond the ones contained in the MSSM, *e.g.* some extra standard model gauge singlets, light stops (predominantly the right-handed ones) and charginos/neutralinos are expected to give quantitatively the same contribution to the final baryon asymmetry.

7.2.1 The electroweak phase transition in the MSSM

As discussed above, a strongly first order electroweak phase transition can be achieved in the presence of a top squark lighter than the top quark [20]. In order to naturally suppress its contribution to the parameter $\Delta\rho$ and hence preserve a good agreement with the precision measurements at LEP, it should be mainly right handed. This can be achieved if the left handed stop soft supersymmetry breaking mass m_Q is much larger than M_Z.

The stop mass matrix is given by

$$\mathcal{M}_{\widetilde{t}} = \left(\begin{array}{cc} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^{*2} & M_{RR}^2 \end{array} \right), \tag{249}$$

where

$$\begin{aligned} M_{LL}^2 &\simeq m_Q^2 + h_t^2 \left| H_2^0 \right|^2, \\ M_{RR}^2 &\simeq m_U^2 + h_t^2 \left| H_2^0 \right|^2, \\ M_{LR}^2 &= h_t \left(A_t H_2^0 - \mu^* H_1^0 \right). \end{aligned} \tag{250}$$

For moderate mixing, the lightest stop mass is then approximately given by

$$m_{\widetilde{t}}^2 \simeq m_U^2 + m_t^2(\phi) \left(1 - \frac{\left| \widetilde{A}_t \right|^2}{m_Q^2} \right) \tag{251}$$

where $\widetilde{A}_t = A_t - \mu^*/\tan\beta$ is the particular combination appearing in the off-diagonal terms of the left-right stop squared mass matrix and m_U^2 is the soft supersymmetry breaking squared mass parameter of the right handed stop. Notice that the Higgs sector contains two neutral CP even states, H_1^0 and H_2^0. However, in the limit in which $m_A \gg T_c$, where m_A is the mass of the pseudoscalar particle of the Higg sector, only one neutral Higgs survives

$$\phi = \cos\beta H_1^0 + \sin\beta H_2^0, \tag{252}$$

where $\tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$, and the low-energy potential reduces to the one-dimensional SM-like potential $V(\phi)$.

The preservation of the baryon number asymmetry requires the order parameter $\langle\phi(T_c)\rangle/T_c$ to be larger than one. The latter is bounded from above

$$\frac{\langle\phi(T_c)\rangle}{T_c} < \left(\frac{\langle\phi(T_c)\rangle}{T_c}\right)_{\rm SM} + \frac{2\, m_t^3 \left(1 - \widetilde{A}_t^2/m_Q^2\right)^{3/2}}{\pi\, v\, m_h^2}\ , \tag{253}$$

where $m_t = \overline{m}_t(m_t)$ is the on-shell running top quark mass in the $\overline{\rm MS}$ scheme. The first term on the right hand side of expression (253) is the Standard Model contribution

$$\left(\frac{\langle\phi(T_c)\rangle}{T_c}\right)_{\rm SM} \simeq \left(\frac{40}{m_h[\rm GeV]}\right)^2, \tag{254}$$

and the second term is the contribution that would be obtained if the right handed stop plasma mass vanished at the critical temperature (see Eq. (255)). Remember that in the expression for the one-loop effective potential (231), the parameter E gets contributions from boson fields. So, the difference between the SM and the MSSM is that light stops may give a large contributions to the effective potential in the MSSM.

In order to overcome the Standard Model constraints, the stop contribution must be therefore large. The stop contribution strongly depends on the value of m_U^2, which must be small in magnitude, and negative, in order to induce a sufficiently strong first order phase transition. Indeed, large stop contributions are always associated with small values of the right handed stop plasma mass

$$m_{\widetilde{t}}^{\rm eff} = -\widetilde{m}_U^2 + \Pi_R(T), \tag{255}$$

where $\widetilde{m}_U^2 = -m_U^2$, $\Pi_R(T) \simeq 4g_3^2T^2/9 + h_t^2/6[2 - \widetilde{A}_t^2/m_Q^2]T^2$ is the finite temperature self-energy contribution to the right-handed squarks. Moreover, the trilinear mass term, $\widetilde{A}_t$, must be $\widetilde{A}_t^2 \ll m_Q^2$ in order to avoid the suppression of the stop contribution to $\langle\phi(T_c)\rangle/T_c$.

Although large values of $\widetilde{m}_U$, of order of the critical temperature, are useful to get a strongly first order phase transition, they may also induce charge and color breaking minima. Indeed, if the effective plasma mass at the critical temperature vanished, the universe would be driven to a charge and color breaking minimum at $T \geq T_c$. Hence, the upper bound on $\langle\phi(T_c)\rangle/T_c$, Eq. (253) cannot be reached in realistic scenarios. A conservative bound on $\widetilde{m}_U$ may be obtained by demanding that the electroweak symmetry breaking minimum should be lower than any color-breaking minima induced by the presence of $\widetilde{m}_U$ at zero temperature, which yields the condition

$$\widetilde{m}_U \leq \left(\frac{m_H^2 v^2 g_3^2}{12} \right)^{1/4} . \tag{256}$$

It can be shown that this condition is sufficient to prevent dangerous color breaking minima at zero and finite temperature for any value of the mixing parameter $\widetilde{A}_t$. A more general analysis is provided in [20].

In order to obtain values of $\langle\phi(T_c)\rangle/T_c$ larger than one, the Higgs mass must take small values, close to the present experimental bound. Numerically, an upper bound, of order 80 GeV, can be derived. For small mixing, the one-loop Higgs mass has a very simple form

$$m_h^2 = M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{\overline{m}_t^4}{v^2} \log\left(\frac{m_{\tilde{t}}^2 m_{\widetilde{T}}^2}{\overline{m}_t^4} \right) \left[1 + \mathcal{O}\left(\frac{\widetilde{A}_t^2}{m_Q^2} \right) \right] , \tag{257}$$

where $m_{\widetilde{T}}^2 \simeq m_Q^2 + m_t^2$, is the heaviest stop squared mass. Hence, $\tan\beta$ must take values close to one. The larger the left handed stop mass, the closer to one $\tan\beta$ must be. This implies that the left handed stop effects decouple at the critical temperature and hence, different values of m_Q mainly affect the baryon asymmetry through the resulting Higgs mass.

Values of the CP-odd Higgs mass $m_A \lesssim 200$ GeV are associated with a weaker first order phase transition. Fig. 11 shows the behaviour of the order parameter $\langle\phi(T_c)\rangle/T_c$ in the m_A-$\tan\beta$ plane, for $\widetilde{A}_t = 0$, $m_Q = 500$ GeV and values of $\widetilde{m}_U$ close to its upper bound, Eq. (256).

In order to correctly interpret the results of Fig. 11 one should remember that the Higgs mass bounds are somewhat weaker for values of $m_A \lesssim 150$ GeV. However, even for values of m_A of order 80 GeV, in the low $\tan\beta$ regime the lower bound on the Higgs mass is of order 60 GeV. Hence, it follows from Fig. 11 that, to obtain a sufficiently strong first order phase transition the CP-odd Higgs mass $m_A \gtrsim 150$ GeV. When two-loop QCD corrections [43, 44] associated with stop loops are included, one finds that $m_A \gtrsim 120$ GeV $m_h \lesssim 85$ GeV. This region will be explored at LEP2 very soon.

Exercise 5

Obtain Eq. (253).

7.2.2 How to produce the baryon asymmetry in the MSSM

As we have previously learned, the first step in the computation of the baryon number asymmetry is to identify those charges which are approximately conserved in the symmetric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, so that the generation of a non-zero baryon charge is energetically favoured according to the Master equation (218).

Charges with these characteristics in the MSSM are the axial stop charge and the Higgsino charge, which may be produced from the interactions of squarks and charginos and/or

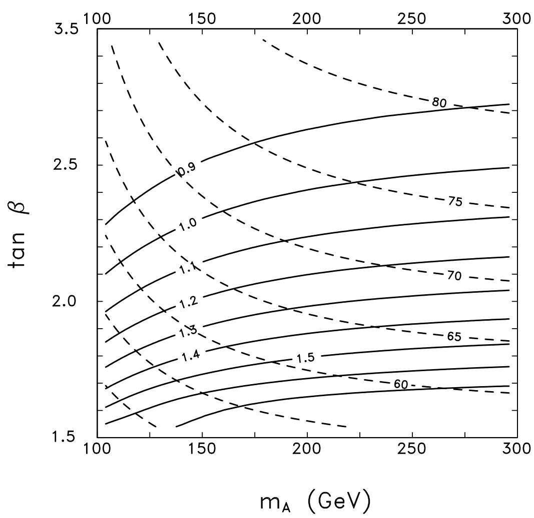


Figure 11: Contour plots of constant values of $\langle\phi(T_c)\rangle/T_c$ (solid lines) and m_h in GeV (dashed lines) in the plane $(m_A, \tan\beta)$. We have fixed $m_t = 175$ GeV and the values of sypersymmetric parameters: $m_Q = 500$ GeV, $m_U = m_U^{\rm crit}$ fixed by the charge and color breaking constraint, and $A_t = \mu/\tan\beta$.

neutralinos with the bubble wall, provided a source of CP-violation is present in these sectors. This is exactly the case, since both the parameters A_t and μ may carry a physical phase [4]. The idea is that, if nonvanishing CP violating sources for the right-handed stop and higgsino numbers are induced in the bubble wall, the scattering among particles as well as diffusion will generate an asymmetry in the left-handed fermion asymmetries in the unbroken phase. The asymmetry – in turn – will fuel baryogenesis because sphaleron transitions will push the system towards the state of minimum free energy, which is the one with nonvanishing baryon asymmetry. In the next subsection, we will give some indications of how to compute the CP-violating sources. Let us now investigate the dynamics of electroweak baryogenesis a little bit further.

One has to start with a set of coupled differential equations describing the effects of diffusion, particle number changing reactions and CP-violating source terms. Major simplifications of the diffusion equations take place when neglecting all the couplings except for gauge interactions and the top Yukawa coupling. Neglecting the weak sphalerons (in the first step) allows to forget about leptons in the diffusion equations and will turn out to be a good approximation when computing Higgs and quark densities.

If the system is near thermal equilibrium and particles interact weakly, the particle number densities n_i may be expressed as (see Eq. (11)) $n_i = k_i \mu_i T^2/6$ where μ_i is the local chemical potential, and k_i are statistical factors of the order of 2 (1) for light bosons (fermions) in thermal equilibrium, and Boltzmann suppressed for particles heavier than T.

What really determines which are the interactions in equilibrium is the typical time scale for the passage of the bubble wall through a given point, $\tau_\omega \sim L_\omega/v_\omega$. If interactions are faster than τ_ω they are in equilibrium, otherwise, they are not.

The particle densities we need to include are

– the left-handed top doublet $q \equiv (t_L + b_L)$,

– the right-handed top quark $t \equiv t_R$,

– the Higgs particle $h \equiv (H_1^0, H_2^0, H_1^-, H_2^+)$, and the superpartners $\widetilde{q}$, $\widetilde{t}$ and $\widetilde{h}$.

The interactions able to change the particle numbers are

– the top Yukawa interaction with rate Γ_t,

– the top quark mass interaction with rate Γ_m,

– the Higgs self-interactions in the broken phase with rate Γ_h,

– the strong sphaleron interactions with rate $\Gamma_{\rm ss}$. The axial vector current of QCD $\sum_i \overline{q}^i \gamma^\mu \gamma_5 q^i = -\sum_i \overline{q}^i_L \gamma^\mu q^i_L + \sum_i \overline{q}^i_R \gamma^\mu q^i_R$ where the sum is over the quarks, has a triangle anomaly and therefore one may expect axial charge violation due to topological transitions analogous to the case of sphaleron transitions

$$\frac{dQ_5}{dt} = -\frac{12 \cdot 6}{T^3}\, \Gamma_{\rm ss}\, Q_5, \tag{258}$$

where Q_5 is the axial charge, the factor 12 comes from the total number of quark chirality states and the factor 6 from the relation between the asymmetry in the quark number density and the chemical potential, $n_i \sim \mu_i T^2/6$, see Eq. (11). The rate of these processes at high temperature is expected to be [92]

$$\Gamma_{\rm ss} = \frac{8}{3}\left(\frac{\alpha_S}{\alpha_W}\right)\Gamma_{\rm sp}, \tag{259}$$

where α_S is the strong fine structure leading to the characteristic time of order of

$$\tau_{\rm ss} \simeq \frac{1}{192\,\kappa\,\alpha_S\,T} \lesssim \tau_\omega. \tag{260}$$

The effect of QCD sphalerons may be represented by the operator

$$\Pi_{i=1}^{3}\,(u_L\,u_R^\dagger\,d_L\,d_R^\dagger)_i, \tag{261}$$

where i is the generation index. Assuming that these processes are in equilibrium [95, 52], we get the following equation for the chemical potentials

$$\sum_{i=1}^{3}(\mu_{u_L^i} - \mu_{u_R^i} + \mu_{d_L^i} - \mu_{d_R^i}) = 0. \tag{262}$$

This equation contains the chemical potential for all the quarks and imposes that the total right-handed baryon number is equal to the total left-handed one. In other words, including strong QCD sphalerons allow the generation of the right-handed bottom quark as well as the generation of the first and second family quarks,

– the weak anomalous interactions with rate $\Gamma_{\rm sp}$,

– the gauge interactions.

We shall assume that the supergauge interactions are in equilibrium, that is

$$\frac{q}{k_q} = \frac{\widetilde{q}}{k_{\widetilde{q}}} = \frac{t}{k_t} = \frac{\widetilde{t}}{k_{\widetilde{t}}} = \frac{h}{k_h} = \frac{\widetilde{h}}{k_{\widetilde{h}}}. \tag{263}$$

Under these assumptions the system may be described by the densities $Q = q + \widetilde{q}$, $T = t + \widetilde{t}$ and $H = h + \widetilde{h}$. CP-violating interactions with the advancing bubble wall produce source terms $\gamma_{\widetilde{H}}$ for Higgsinos and γ_R for right-handed stops, which tend to push the system out of equilibrium. Ignoring the curvature of the bubble wall, any quantity becomes a function of the coordinate $\mathbf{z} = z_3 + v_\omega z$, the coordinate normal to the wall surface, where we assume the bubble wall is moving along the z_3-axis.

When including the strong sphalerons, right-handed bottom quarks are generated as well as the quarks of the first two families. However, since strong sphalerons are the only processes which produce the first two generation quarks and all quarks have nearly the same diffusion constants, we may constrain the densities algebrically in terms of $B \equiv b_R + \widetilde{b}_R$

$$Q_{1L} = Q_{2L} = -2U_R = -2D_R = -2S_R = -2C_R = -2B = 2(Q+T), \tag{264}$$

where the last equality comes from imposing that strong sphalerons are in equilibrium.

Particle transport is treated by including a diffusion term. Taking all the quarks and squarks with the same diffusion constant D_q and Higgs and Higgsinos with diffusion constant D_h, onecan write the folowing set of diffusion equations

$$\begin{aligned}
\dot{Q} &= D_q\nabla^2 Q - \Gamma_t[Q/k_Q - H/k_H - T/k_T] - \Gamma_m[Q/k_Q - T/k_T] \\
&- 6\Gamma_{\rm ss}[2Q/k_Q - T/k_T + 9(Q+T)/k_B] + \gamma_{\widetilde{t}}, \\
\dot{T} &= D_q\nabla^2 T - \Gamma_t[-Q/k_Q + H/k_H + T/k_T] - \Gamma_m[-Q/k_Q + T/k_T] \\
&+ 3\Gamma_{\rm ss}[2Q/k_Q - T/k_T + 9(Q+T)/k_B] + \gamma_{\widetilde{t}}, \\
\dot{H} &= D_h\nabla^2 h - \Gamma_t[-Q/k_Q + H/k_H + T/k_T] - \Gamma_h H/k_H + \gamma_{\widetilde{h}},
\end{aligned} \tag{265}$$

where we have inserted the CP violating sources.

Assuming that the rates Γ_t and $\Gamma_{\rm ss}$ are fast so that $Q/k_q - H/k_H - T/k_T = \mathcal{O}(1/\Gamma_t)$ and $2Q/k_q - T/k_T + 9(Q+T)/k_b = \mathcal{O}(1/\Gamma_{\rm ss})$, one can find the equation governing the Higgs density

$$v_\omega H' - \overline{D} H'' + \overline{\Gamma} H - \widetilde{\gamma} = 0, \tag{266}$$

where the derivatives are now with respect to $\mathbf{z}$, $\overline{D}$ is the effective diffusion constant, $\widetilde{\gamma}$ is an effective source term in the frame of the bubble wall and $\overline{\Gamma}$ is the effective decay constant [102]. An analytical solution to Eq. (266) satisfying the boundary conditions $H(\pm\infty) = 0$ may be found in the symmetric phase (defined by $\mathbf{z} < 0$) using a $\mathbf{z}$-independent effective diffusion constant and a step function for the effective decay rate $\overline{\Gamma} = \widetilde{\Gamma}\theta(\mathbf{z})$. A more realistic form of $\overline{\Gamma}$ would interpolate smoothly between the symmetric and the broken phase values. The values of $\overline{D}$ and $\overline{\Gamma}$ in (266) of course depend on the particular values of supersymmetric parameters. For the considered range one typically finds $\overline{D} \sim 0.8$ GeV^{-1}, $\overline{\Gamma} \sim 1.7$ GeV.

The tunneling processes from the symmetric phase to the true minimum in the first order phase transition of the Higgs field in the MSSM has been recently analyzed in [99] including the leading two-loop effects. It was shown that the Higgs profile along the bubbles at the time when the latter are formed has a typical thickness $L_\omega \sim (20-30)/T$. In general, however, the value of L_ω when the bubbles are moving through the plasma with some velocity v_ω is different from the value at bubble nucleation. Indeed, the motion of the bubble wall is determined by two main factors, namely the pressure difference between inside and outside the bubble –leading to the expansion– and the friction force, proportional to v_ω, accounting for the collisions of the plasma particles off the wall. The equilibrium between these two forces imples a steady state with a final velocity v_ω. If bubbles are rather thick, thermodinamical conditions are established inside the wall and for the latter is no longer possible to loose energy by thermal dissipation. Under these conditions the bubble wall is accelerated until slightly out-of-equilibrium conditions and the friction forces are reestablished. As we shall see, the total amount of the baryon asymmetry is proportional to $\Delta\beta$ –the change in the ratio of the Higgs vacuum expectation values $\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$ from $\mathbf{z} = 0$ to inside the bubble wall. This quantity tends to zero for large values of m_A, and takes small values, of order 10^{-2} for values of the pseudoscalar mass $m_A =$ 150–200 GeV [99].

The solution of Eq. (266) for $\mathbf{z} < 0$ is

$$H(\mathbf{z}) = \mathcal{A}\, {\rm e}^{\mathbf{z} v_\omega/\overline{D}}, \tag{267}$$

and for $\mathbf{z} > 0$ is

$$\begin{aligned} H(\mathbf{z}) &= \left(\mathcal{B}_+ - \frac{1}{\overline{D}(\lambda_+ - \lambda_-)}\int_0^{\mathbf{z}} du \widetilde{\gamma}(u) e^{-\lambda_+ u}\right) e^{\lambda_+ \mathbf{z}} \\ &+ \left(\mathcal{B}_- - \frac{1}{\overline{D}(\lambda_- - \lambda_+)}\int_0^{\mathbf{z}} du \widetilde{\gamma}(u) e^{-\lambda_- u}\right) e^{\lambda_- \mathbf{z}}. \end{aligned} \tag{268}$$

where

$$\lambda_\pm = \frac{v_\omega \pm \sqrt{v_\omega^2 + 4\widetilde{\Gamma}\overline{D}}}{2\overline{D}}. \tag{269}$$

Imposing the continuity of H and H' at the boundaries, we find

$$\mathcal{A} = \mathcal{B}_+ \left(1 - \frac{\lambda_-}{\lambda_+}\right) = \mathcal{B}_- \left(\frac{\lambda_+}{\lambda_-} - 1\right) = \frac{1}{\overline{D}\,\lambda_+} \int_0^\infty du\, \widetilde{\gamma}(u) e^{-\lambda_+ u}. \tag{270}$$

From the form of the above equations one can see that CP violating densities are non zero for a time $t \sim \overline{D}/v_\omega^2$ and the assumptions leading to the analytical form of $H(\mathbf{z})$ are valid provided that the interaction rates Γ_t and $\Gamma_{\rm ss}$ are larger than $v_\omega^2/\overline{D}$ [102, 19].

The equation governing the baryon asymmetry n_B is given by [102]

$$D_q n_B'' - v_\omega n_B' - \theta(-\mathbf{z}) N_f \Gamma_{\rm sp} n_L = 0, \tag{271}$$

where n_L is the total number density of left-handed weak doublet fermions and we have assumed that the baryon asymmetry gets produced only in the symmetric phase. Expressing $n_L(\mathbf{z})$ in terms of the Higgs number density

$$n_L = \frac{9k_q k_T - 8k_b k_T - 5k_b k_q}{k_H(k_b + 9k_q + 9k_T)}\, H \tag{272}$$

and making use of Eqs. (267)-(271), we find that

$$\frac{n_B}{s} = -g(k_i) \frac{\mathcal{A}\overline{D}\Gamma_{\rm sp}}{v_\omega^2 s}, \tag{273}$$

where $g(k_i)$ is a numerical coefficient depending upon the light degrees of freedom present in the thermal bath.

Eq. (273) summarizes all the ingredients we need to produce a baryon asymmetry in electroweak baryogenesis: *1)* (the integral of) a CP violating source $\mathcal{A}$, *2)* baryon number violation provided by the sphaleron transitions with rate $\Gamma_{\rm sp}$ and *3)* out-of-equilibrium conditions provided by the expanding bubble wall.

7.2.3 Out-of-equilibrium field theory with a broad brush

The next step in the computation of the baryon asymmetry is the evaluation of the CP violating sources for the right-handed stop number and the higgsino number.

Non-equilibrium Quantum Field Theory provides us with the necessary tools to write down a set of quantum Boltzmann equations (QBE's) describing the local particle densities and automatically incorporating the CP violating sources. The most appropriate extension of the field theory to deal with these issues is to generalize the time contour of integration to a closed time-path (CTP). The CTP formalism is a powerful Green's function formulation for describing non-equilibrium phenomena in field theory, it leads to a complete non-equilibrium quantum kinetic theory approach and to a rigorous computation of the CP violating sources for the stop and the Higgsino numbers [109, 110, 111]. What is more relevant, though, is that the CP violating sources– and more generally the particle number changing interactions– built up from the CTP formalism are characterized by "memory" effects which are typical of the quantum transport theory [32, 58]. CP violating sources are built up when right-handed stops and Higgsinos scatter off the advancing Higgs bubble wall and CP is violated at the vertices of interactions. In the classical kinetic theory the "scattering term" does

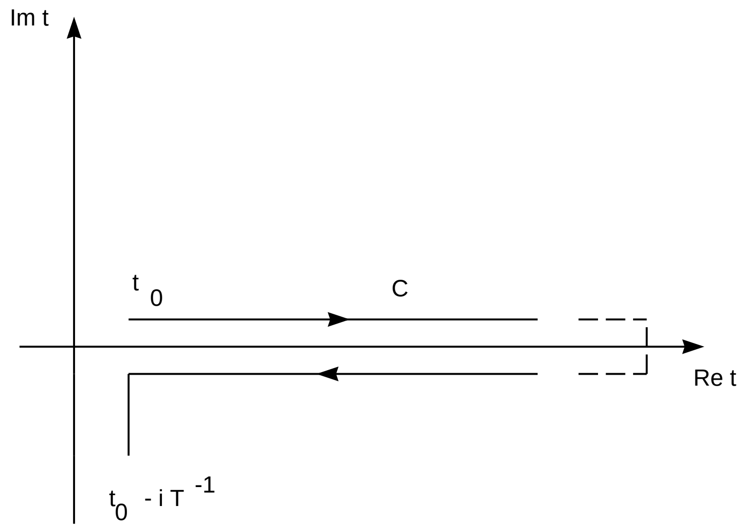


Figure 12: The contour C in the $(\mathrm{Re}\, t, \mathrm{Im}\, t)$ plane proper to the CTP formalism

not include any integral over the past history of the system. This is equivalent to assuming that any collision in the plasma does not depend upon the previous ones. On the contrary, the quantum approach reveals that the CP violating source is manifestly non-Markovian.

We will now briefly present some of the basic features of the non-equilibrium quantum field theory based on the Schwinger-Keldysh formulation [115, 61]. The interested reader is referred to the excellent review by Chou *et* al. [22] for a more comprehensive discussion.

Since we need the temporal evolution of the particle asymmetries with definite initial conditions and not simply the transition amplitude of particle reactions, the ordinary equilibrium quantum field theory at finite temperature is not the appropriate tool. The most appropriate extension of the field theory to deal with nonequilibrium phenomena amounts to generalize the time contour of integration to a closed-time path. More precisely, the time integration contour is deformed to run from $-\infty$ to $+\infty$ and back to $-\infty$, see Fig. 12.

The CTP formalism (often dubbed as in-in formalism) is a powerful Green's function formulation for describing non-equilibrium phenomena in field theory. It allows to describe phase-transition phenomena and to obtain a self-consistent set of quantum Boltzmann equations. The formalism yields various quantum averages of operators evaluated in the in-state without specifying the out-state. On the contrary, the ordinary quantum field theory (often dubbed as in-out formalism) yields quantum averages of the operators evaluated with an in-state at one end and an out-state at the other.

Because of the time contour deformation, the partition function in the in-in formalism for a *complex* scalar field is defined to be

$$Z\left[J, J^\dagger\right] \;=\; \mathrm{Tr}\left[T\left(\exp\left[i\int_{\mathrm{C}}\left(J\phi + J^\dagger\phi^\dagger\right)\right]\right)\rho\right]$$

$$
\begin{aligned}
&= \mathrm{Tr}\left[T_+\left(\exp\left[i\int\left(J_+\phi_+ + J_+^\dagger\phi_+^\dagger\right)\right]\right)\right. \\
&\times\left. T_-\left(\exp\left[-i\int\left(J_-\phi_- + J_-^\dagger\phi_-^\dagger\right)\right]\right)\rho\right], \qquad (274)
\end{aligned}
$$

where the suffic C in the integral denotes that the time integration contour runs from minus infinity to plus infinity and then back to minus infinity again. The symbol ρ represents the initial density matrix and the fields are in the Heisenberg picture and defined on this closed time contour. As with the Euclidean time formulation, scalar (fermionic) fields ϕ are still periodic (anti-periodic) in time, but with $\phi(t,\vec{x}) = \phi(t-i\beta,\vec{x})$, $\beta = 1/T$. The temperature appears due to boundary condition, but time is now explicitly present in the integration contour.

We must now identify field variables with arguments on the positive or negative directional branches of the time path. This doubling of field variables leads to six different real-time propagators on the contour [22]. These six propagators are not independent, but using all of them simplifies the notation. For a generic bosonic charged scalar field ϕ they are defined as

$$
\begin{aligned}
G^>_\phi(x,y) &= -i\langle\phi(x)\phi^\dagger(y)\rangle, \\
G^<_\phi(x,y) &= -i\langle\phi^\dagger(y)\phi(x)\rangle, \\
G^t_\phi(x,y) &= \theta(x,y)G^>_\phi(x,y) + \theta(y,x)G^<_\phi(x,y), \\
G^{\bar{t}}_\phi(x,y) &= \theta(y,x)G^>_\phi(x,y) + \theta(x,y)G^<_\phi(x,y), \\
G^r_\phi(x,y) &= G^t_\phi - G^<_\phi = G^>_\phi - G^{\bar{t}}_\phi, \quad G^a_\phi(x,y) = G^t_\phi - G^>_\phi = G^<_\phi - G^{\bar{t}}_\phi, \qquad (275)
\end{aligned}
$$

where the last two Green functions are the retarded and advanced Green functions respectively and $\theta(x,y) = \theta(t_x - t_y)$ is the step function. For a generic fermion field ψ the six different propagators are analogously defined as

$$
\begin{aligned}
G^>_\psi(x,y) &= -i\langle\psi(x)\bar{\psi}(y)\rangle, \\
G^<_\psi(x,y) &= +i\langle\bar{\psi}(y)\psi(x)\rangle, \\
G^t_\psi(x,y) &= \theta(x,y)G^>_\psi(x,y) + \theta(y,x)G^<_\psi(x,y), \\
G^{\bar{t}}_\psi(x,y) &= \theta(y,x)G^>_\psi(x,y) + \theta(x,y)G^<_\psi(x,y), \\
G^r_\psi(x,y) &= G^t_\psi - G^<_\psi = G^>_\psi - G^{\bar{t}}_\psi, \quad G^a_\psi(x,y) = G^t_\psi - G^>_\psi = G^<_\psi - G^{\bar{t}}_\psi. \qquad (276)
\end{aligned}
$$

For equilibrium phenomena, the brackets $\langle\cdots\rangle$ imply a thermodynamic average over all the possible states of the system. While for homogeneous systems in equilibrium, the Green functions depend only upon the difference of their arguments $(x,y) = (x-y)$ and there is no dependence upon $(x+y)$, for systems out of equilibrium, the definitions (275) and (276) have a different meaning. The concept of thermodynamic averaging is now ill-defined. Instead, the bracket means the need to average over all the available states of the system for the non-equilibrium distributions. Furthermore, the arguments of the Green functions (x,y) are *not* usually given as the difference $(x-y)$. For example, non-equilibrium could be caused by transients which make the Green functions depend upon (t_x, t_y) rather than $(t_x - t_y)$.

For interacting systems whether in equilibrium or not, one must define and calculate self-energy functions. Again, there are six of them: Σ^t, $\Sigma^{\bar{t}}$, $\Sigma^<$, $\Sigma^>$, Σ^r and Σ^a. The same relationships exist among them as for the Green functions in (275) and (276), such as

$$\Sigma^r = \Sigma^t - \Sigma^< = \Sigma^> - \Sigma^{\bar{t}}, \quad \Sigma^a = \Sigma^t - \Sigma^> = \Sigma^< - \Sigma^{\bar{t}}. \tag{277}$$

The self-energies are incorporated into the Green functions through the use of Dyson's equations. A useful notation may be introduced which expresses four of the six Green functions as the elements of two-by-two matrices [31]

$$\widetilde{G} = \begin{pmatrix} G^t & \pm G^< \\ G^> & -G^{\bar{t}} \end{pmatrix}, \quad \widetilde{\Sigma} = \begin{pmatrix} \Sigma^t & \pm\Sigma^< \\ \Sigma^> & -\Sigma^{\bar{t}} \end{pmatrix}, \tag{278}$$

where the upper signs refer to bosonic case and the lower signs to fermionic case. For systems either in equilibrium or non-equilibrium, Dyson's equation is most easily expressed by using the matrix notation

$$\widetilde{G}(x,y) = \widetilde{G}^0(x,y) + \int d^4x_3 \int d^4x_4\, \widetilde{G}^0(x,x_3)\widetilde{\Sigma}(x_3,x_4)\widetilde{G}(x_4,y), \tag{279}$$

where the superscript "0" on the Green functions means to use those for *noninteracting* system. This equation appears quite formidable; however, some simple expressions may be obtained for the respective Green functions. It is useful to notice that Dyson's equation can be written in an alternate form, instead of (279), with $\widetilde{G}^0$ on the right in the interaction terms,

$$\widetilde{G}(x,y) = \widetilde{G}^0(x,y) + \int d^4x_3 \int d^4x_4\, \widetilde{G}(x,x_3)\widetilde{\Sigma}(x_3,x_4)\widetilde{G}^0(x_4,y). \tag{280}$$

Equations. (279) and (280) are the starting points to derive the quantum Boltzmann equations describing the temporal evolution of the CP violating particle density asymmetries.

7.2.4 The quantum Boltzmann equations

Our goal now is to find the QBE for the generic bosonic CP violating current

$$\langle J^\mu_\phi(x)\rangle \equiv i\langle\phi^\dagger(x) \overset{\leftrightarrow\mu}{\partial_x} \phi(x)\rangle \equiv \left[n_\phi(x), \vec{J}_\phi(x)\right]. \tag{281}$$

The zero-component of this current n_ϕ represents the number density of particles minus the number density of antiparticles and is therefore the quantity which enters the diffusion equations of supersymmetric electroweak baryogenesis.

Since the CP violating current can be expressed in terms of the Green function $G^<_\phi(x,y)$ as

$$\langle J^\mu_\phi(x)\rangle = -\left(\partial^\mu_x - \partial^\mu_y\right) G^<_\phi(x,y)\Big|_{x=y}, \tag{282}$$

the problem is reduced to find the QBE for the interacting Green function $G^<_\phi(x,y)$ when the system is not in equilibrium. This equation can be found from (279) by operating by $\left(\vec{\Box}_x + m^2\right)$ on both sides of the equation. Here m represents the bare mass term of the field ϕ. On the right-hand side, this operator acts only on $\widetilde{G}^0_\phi$

$$\left(\vec{\Box}_x + m^2\right) \widetilde{G}_\phi(x,y) = \delta^{(4)}(x,y)\widetilde{I}_4 + \int d^4x_3 \widetilde{\Sigma}_\phi(x,x_3)\widetilde{G}_\phi(x_3,y), \tag{283}$$

where I is the identity matrix. It is useful to also have an equation of motion for the other variable y. This is obtained from (280) by operating by $\left(\overleftarrow{\Box}_y + m^2\right)$ on both sides of the equation. We obtain

$$\widetilde{G}_\phi(x,y)\left(\overleftarrow{\Box}_y + m^2\right) = \delta^{(4)}(x,y)\widetilde{I}_4 + \int d^4x_3 \widetilde{G}_\phi(x,x_3)\widetilde{\Sigma}_\phi(x_3,y). \tag{284}$$

The two equations (283) and (284) are the starting point for the derivation of the QBE for the particle asymmetries. Let us extract from (283) and (284) the equations of motions for the Green function $G^{<}_\phi(x,y)$

$$\begin{aligned}\left(\overrightarrow{\Box}_x + m^2\right) G^{<}_\phi(x,y) &= \int d^4x_3 \left[\Sigma^t_\phi(x,x_3)G^{<}_\phi(x_3,y) - \Sigma^{<}_\phi(x,x_3)G^{\bar{t}}_\phi(x_3,y)\right], &(285)\\ G^{<}_\phi(x,y)\left(\overleftarrow{\Box}_y + m^2\right) &= \int d^4x_3 \left[G^t_\phi(x,x_3)\Sigma^{<}_\phi(x_3,y) - G^{<}_\phi(x,x_3)\Sigma^{\bar{t}}_\phi(x_3,y)\right]. &(286)\end{aligned}$$

If we now substract the two equations and make the identification $x = y$, the left-hand side is given by

$$\partial^x_\mu \left[\left(\partial^\mu_x - \partial^\mu_y\right) G^{<}_\phi(x,y)\right]\Big|_{x=y} = -\frac{\partial J^\mu_\phi(X)}{\partial X^\mu} = -\left(\frac{\partial n_\phi}{\partial T} + \vec{\nabla}\cdot\vec{j}_\phi\right), \tag{287}$$

and the QBE for the particle density asymmetry is therefore obtained to be

$$\frac{\partial n_\phi(X)}{\partial T} + \vec{\nabla}\cdot\vec{j}_\phi(X) = -\int d^4x_3 \left[\Sigma^t_\phi G^{<}_\phi - \Sigma^{<}_\phi G^{\bar{t}}_\phi - G^t_\phi\Sigma^{<}_\phi - G^{<}_\phi\Sigma^{\bar{t}}_\phi\right]\Big|_{x=y}, \tag{288}$$

where we have defined the centre-of-mass coordinate system

$$X = (T,\vec{X}) = \frac{1}{2}(x+y), \quad (t,\vec{r}) = x - y. \tag{289}$$

Notice that T now means the centre-of-mass time and not temperature. The identification $x = y$ in Eq. (288) is therefore equivalent to require $t = \vec{r} = 0$.

In order to examine the "scattering term" on the right-hand side of Eq. (288), the first step is to restore all the variable arguments. Setting $x = y$ in the original notation of $\Sigma_\phi(x,x_3)G_\phi(x_3,y)$ gives $(X,x_3)(x_3,X)$ for the pair of arguments

$$\begin{aligned}\frac{\partial n_\phi(X)}{\partial T} + \vec{\nabla}\cdot\vec{j}_\phi(X) = &-\int d^4x_3 \left[\Sigma^t_\phi(X,x_3)G^{<}_\phi(x_3,X) - \Sigma^{<}_\phi(X,x_3)G^{\bar{t}}_\phi(x_3,X)\right.\\ &+ \left. G^t_\phi(X,x_3)\Sigma^{<}_\phi(x_3,X) - G^{<}_\phi(X,x_3)\Sigma^{\bar{t}}_\phi(x_3,X)\right].\end{aligned} \tag{290}$$

The next step is to employ the definitions in (275) to express the time-ordered functions G^t_ϕ, $G^{\bar{t}}_\phi$, Σ^t_ϕ, and $\Sigma^{\bar{t}}_\phi$ in terms of $G^{<}_\phi$, $G^{>}_\phi$, $\Sigma^{<}_\phi$ and $G^{>}_\phi$. Then the time integrals are separated into whether $t_3 > T$ or $t_3 < T$ and the right-hand side of Eq. (290) reads

$$\begin{aligned}= &-\int d^4x_3 \left\{\theta(T-t_3)\left[\Sigma^{>}_\phi G^{<}_\phi + G^{<}_\phi\Sigma^{>}_\phi - \Sigma^{<}_\phi G^{>}_\phi - G^{>}_\phi\Sigma^{<}_\phi\right]\right.\\ &+ \left.\theta(t_3-T)\left[\Sigma^{<}_\phi G^{<}_\phi + G^{<}_\phi\Sigma^{<}_\phi - \Sigma^{<}_\phi G^{<}_\phi - G^{<}_\phi\Sigma^{<}_\phi\right]\right\}.\end{aligned} \tag{291}$$

The term with $t_3 > T$ all cancel, leaving $T > t_3$. Rearranging these terms gives [109, 110, 111]

$$\frac{\partial n_\phi(X)}{\partial T} + \vec{\nabla} \cdot \vec{j}_\phi(X) = -\int d^3\vec{x}_3 \int_{-\infty}^{T} dt_3 \left[\Sigma_\phi^>(X,x_3)G_\phi^<(x_3,X)\right.$$
$$- \left. G_\phi^>(X,x_3)\Sigma_\phi^<(x_3,X) + G_\phi^<(X,x_3)\Sigma_\phi^>(x_3,X) - \Sigma_\phi^<(X,x_3)G_\phi^>(x_3,X)\right]. \quad (292)$$

This equation is the QBE for the particle density asymmetry and it can be explicitly checked that, in the particular case in which interactions conserve the number of particles and the latter are neither created nor destroyed, the number asymmetry n_ϕ is conserved and obeys the equation of continuity $\partial n_\phi/\partial T + \vec{\nabla} \cdot \vec{j}_\phi = 0$. During the production of the baryon asymmetry, however, particle asymmetries are not conserved. This occurs because the interactions themselves do not conserve the particle number asymmetries and there is some source of CP violation in the system. The right-hand side of Eq. (292), through the general form of the self-energy Σ_ϕ, contains all the information necessary to describe the temporal evolution of the particle density asymmetries: particle number changing reactions and CP violating source terms, which will pop out from the corresponding self-energy Σ_{CP}. If the interactions of the system do not violate CP, there will be no CP violating sources and the final baryon asymmetry produced during supersymmetric baryogenesis will be vanishing.

The kinetic Eq. (292) has an obvious interpretation in terms of gain and loss processes. What is unusual, however, is the presence of the integral over the time: the equation is manifestly non-Markovian. Only the assumption that the relaxation time scale of the particle asymmetry is much longer than the time scale of the non-local kernels leads to a Markovian description. A further approximation, *i.e.* taking the upper limit of the time integral to $T \to \infty$, leads to the familiar Boltzmann equation. The physical interpretation of the integral over the past history of the system is straightforward: it leads to the typical "memory" effects which are observed in quantum transport theory [32, 58]. In the classical kinetic theory the "scattering term" does not include any integral over the past history of the system which is equivalent to assume that any collision in the plasma does not depend upon the previous ones. On the contrary, quantum distributions posses strong memory effects and the thermalization rate obtained from the quantum transport theory may be substantially longer than the one obtained from the classical kinetic theory. As shown in [109, 110, 111] , memory effects play a fundamental role in the determination of the CP violating sources which fuel baryogenesis when transport properties allow the CP violating charges to diffuse in front of the bubble wall separating the broken from the unbroken phase at the electroweak phase transition.

Notice that so far we have not made any approximation and the computation is therefore valid for all shapes and sizes of the bubble wall expanding in the thermal bath during a first-order electroweak phase transition.

Let us now focus on the generic fermionic CP violating current. It reads

$$\langle J_\psi^\mu(x)\rangle \equiv \langle\bar{\psi}(x)\gamma^\mu\psi(x)\rangle \equiv \left[n_\psi(x), \vec{J}_\psi(x)\right], \quad (293)$$

where ψ indicates a Dirac fermion and γ^μ represent the usual Dirac matrices. Again, the zero-component of this current n_ψ represents the number density of particles minus the number density of antiparticles and is therefore the relevant quantity for the diffusion equations of supersymmetric electroweak baryogenesis.

We want to find a couple of equations of motion for the interacting fermionic Green function $\widetilde{G}_\psi(x,y)$ when the system is not in equilibrium. Such equations may be found by applying the operators $\left(i \overrightarrow{\not\partial}_x - M\right)$ and $\left(i \overleftarrow{\not\partial}_y + M\right)$ on both sides of Eqs. (279) and (280), respectively. Here M represents the bare mass term of the fermion ψ. We find

$$\begin{aligned}\left(i \overrightarrow{\not\partial}_x - M\right)\widetilde{G}_\psi(x,y) &= \delta^{(4)}(x,y)\widetilde{I}_4 + \int d^4x_3 \widetilde{\Sigma}_\psi(x,x_3)\widetilde{G}_\psi(x_3,y), && (294)\\ \widetilde{G}_\psi(x,y)\left(i \overleftarrow{\not\partial}_y + M\right) &= -\delta^{(4)}(x,y)\widetilde{I}_4 - \int d^4x_3 \widetilde{G}_\psi(x,x_3)\widetilde{\Sigma}_\psi(x_3,y). && (295)\end{aligned}$$

We can now take the trace over the spinorial indeces of both sides of the equations, sum up the two equations above and finally extract the equation of motion for the Green function $G^{>}_\psi$

$$\begin{aligned}\mathrm{Tr}\left\{\left[i \overrightarrow{\not\partial}_x + i \overleftarrow{\not\partial}_y\right] G^{>}_\psi(x,y)\right\} &= \int d^4x_3 \,\mathrm{Tr}\left[\Sigma^{>}_\psi(x,x_3)G^{t}_\psi(x_3,y) - \Sigma^{\bar{t}}_\psi(x,x_3)G^{>}_\psi(x_3,y)\right.\\ &- \left. G^{>}_\psi(x,x_3)\Sigma^{t}_\psi(x_3,y) + G^{\bar{t}}_\psi(x,x_3)\Sigma^{>}_\psi(x_3,y)\right]. \qquad (296)\end{aligned}$$

Making use of the centre-of-mass coordinate system, we can work out the left-hand side of Eq. (296)

$$\begin{aligned}&\mathrm{Tr}\left[i \overrightarrow{\not\partial}_x G^{>}_\psi(T,\vec{X},t,\vec{r}) + G^{>}_\psi(T,\vec{X},t,\vec{r}) i \overleftarrow{\not\partial}_y\right]\Big|_{t=\vec{r}=0}\\ =\;& i\left(\partial^x_\mu + \partial^y_\mu\right) i\langle\bar{\psi}\gamma^\mu\psi\rangle\Big|_{t=\vec{r}=0}\\ =\;& -\frac{\partial}{\partial X^\mu}\langle\bar{\psi}(X)\gamma^\mu\psi(X)\rangle\\ =\;& -\frac{\partial}{\partial X^\mu}J^\mu_\psi. \qquad (297)\end{aligned}$$

The next step is to employ the definitions in (276) to express the time-ordered functions G^t_ψ, $G^{\bar{t}}_\psi$, Σ^t_ψ, and $\Sigma^{\bar{t}}_\psi$ in terms of $G^{<}_\psi$, $G^{>}_\psi$, $\Sigma^{<}_\psi$ and $G^{>}_\psi$. The computation goes along the same lines as the analysis made in the previous section and we get [109, 110, 111]

$$\begin{aligned}\frac{\partial n_\psi(X)}{\partial T} + \vec{\nabla}\cdot\vec{j}_\psi(X) &= \int d^3\vec{x}_3 \int_{-\infty}^{T} dt_3 \,\mathrm{Tr}\left[\Sigma^{>}_\psi(X,x_3)G^{<}_\psi(x_3,X)\right.\\ - \; G^{>}_\psi(X,x_3)\Sigma^{<}_\psi(x_3,X) &+ G^{<}_\psi(X,x_3)\Sigma^{>}_\psi(x_3,X) - \left.\Sigma^{<}_\psi(X,x_3)G^{>}_\psi(x_3,X)\right]. \qquad (298)\end{aligned}$$

This is the "diffusion" equation describing the temporal evolution of a generic fermionic number asymmetry n_ψ. As for the bosonic case, all the information regarding particle number violating interactions and CP violating sources are stored in the self-energy Σ_ψ.

7.2.5 The CP violating source for higgsinos and the final baryon asymmetry

As we mentioned, a strongly first order electroweak phase transition can be achieved in the presence of a top squark lighter than the top quark. In order to naturally suppress its contribution to the parameter $\Delta\rho$ and hence preserve a good agreement with the precision measurements at LEP, it should be mainly right-handed. This can be achieved if the left-handed stop soft supersymmetry breaking mass m_Q is much larger than M_Z. Under this assumption, however, the right-handed stop contribution to the baryon asymmetry results to be negligible. We will concentrate, therefore, only on the CP violating source for the Higgsino.

The Higgs fermion current associated with neutral and charged Higgsinos can be written as

$$J^{\mu}_{\widetilde{H}} = \overline{\widetilde{H}}\gamma^{\mu}\widetilde{H} \tag{299}$$

where $\widetilde{H}$ is the Dirac spinor

$$\widetilde{H} = \begin{pmatrix} \widetilde{H}_2 \\ \overline{\widetilde{H}}_1 \end{pmatrix} \tag{300}$$

and $\widetilde{H_2} = \widetilde{H}_2^0$ ($\widetilde{H}_2^+$), $\widetilde{H_1} = \widetilde{H}_1^0$ ($\widetilde{H}_1^-$) for neutral (charged) Higgsinos. The processes in the plasma which change the Higgsino number are the ones induced by the top Yukawa coupling and by interactions with the Higgs profile. The interactions among the charginos and the charged Higgsinos which are responsible for the CP violating source in the diffusion equation for the Higgs fermion number read

$$\mathcal{L} = -g_2 \left\{ \overline{\widetilde{H}} \left[v_1(x) P_L + \mathrm{e}^{i\theta_\mu} v_2(x) P_R \right] \widetilde{W} \right\} + \text{h.c.}, \tag{301}$$

where θ_μ is the phase of the μ-parameter and we have indicated $\langle H_i^0(x)\rangle$ by $v_i(x)$, $i = 1, 2$.

Analogously, the interactions among the Bino, the $\widetilde{W}_3$-ino and the neutral Higgsinos are

$$\mathcal{L} = -\frac{1}{2} \left\{ \overline{\widetilde{H^0}} \left[v_1(x) P_L + \mathrm{e}^{i\theta_\mu} v_2(x) P_R \right] \left(g_2 \widetilde{W}_3 - g_1 \widetilde{B} \right) \right\} + \text{h.c.} \tag{302}$$

To compute the source for the Higgs fermion number $\gamma_{\widetilde{H}}$ we perform a "Higgs insertion expansion" around the symmetric phase. At the lowest level of perturbation, the interactions of the charged Higgsino induce a contribution to the self-energy of the form (and analogously for the other component $\delta\Sigma^{\mathrm{CP},>}_{\widetilde{H}}$)

$$\delta\Sigma^{CP,<}_{\widetilde{H}}(x,y) = g^L_{CP}(x,y) P_L G^{0,<}_{\widetilde{W}}(x,y) P_L + g^R_{CP}(x,y) P_R G^{0,<}_{\widetilde{W}}(x,y) P_R, \tag{303}$$

where

$$\begin{aligned} g^L_{CP}(x,y) &= g_2^2 v_1(x) v_2(y) \mathrm{e}^{-i\theta_\mu}, \\ g^R_{CP}(x,y) &= g_2^2 v_1(y) v_2(x) \mathrm{e}^{i\theta_\mu}. \end{aligned} \tag{304}$$

We have approximated the exact Green function of winos $G_{\widetilde{W}}$ by the equilibrium Green function in the unbroken phase $G^0_{\widetilde{W}}$. This is because any departure from thermal equilibrium distribution functions is caused at a given point by the passage of the wall and, therefore, is $\mathcal{O}(v_\omega)$. Since we will show that the source is already linear in v_ω, working with thermal *equilibrium* Green functions in the unbroken phase amounts to ignoring terms of higher order in v_ω. This is accurate as long as the bubble wall is moving slowly in the plasma. Similar formulae hold for the neutral Higgsinos.

The dispersion relations of charginos and neutralinos are changed by high temperature corrections [123]. Even though fermionic dispersion relations are highly nontrivial, especially when dealing with Majorana fermions [107], relatively simple expressions for the equilibrium fermionic spectral functions may be given in the limit in which the damping rate is smaller than the typical self-energy of the fermionic excitation [58]. If we now insert the expressions (303) and (304) into the QBE (298), we get the CP violating source [109, 110, 111]

$$\begin{aligned}\gamma_{\widetilde{H}} &= -\int d^3\vec{x}_3 \int_{-\infty}^{T} dt_3 \,\mathrm{Tr}\left[\delta\Sigma^{CP,>}_{\widetilde{H}}(X,x_3)G^{0,<}_{\widetilde{H}}(x_3,X) - G^{0,>}_{\widetilde{H}}(X,x_3)\delta\Sigma^{CP,<}_{\widetilde{H}}(x_3,X)\right. \\ &+ \left. G^{0,<}_{\widetilde{H}}(X,x_3)\delta\Sigma^{CP,>}_{\widetilde{H}}(x_3,X) - \delta\Sigma^{CP,<}_{\widetilde{H}}(X,x_3)G^{0,>}_{\widetilde{H}}(x_3,X)\right], \end{aligned} \tag{305}$$

which contains in the integrand the following function

$$g^L_{CP}(X,x_3)+g^R_{CP}(X,x_3)-g^L_{CP}(x_3,X)-g^R_{CP}(x_3,X) = 2i\sin\theta_\mu\left[v_2(X)v_1(x_3) - v_1(X)v_2(x_3)\right], \tag{306}$$

which vanishes if $\mathrm{Im}(\mu)=0$ and if the $\tan\beta(x)$ is a constant along the Higgs profile.

In order to deal with analytic expressions, we can work out the thick wall limit and simplify the expressions obtained above by performing a derivative expansion

$$v_i(x_3) = \sum_{n=0}^{\infty}\frac{1}{n!}\,\frac{\partial^n}{\partial(X^\mu)^n}v_i(X)\,(x_3^\mu - X^\mu)^n\,. \tag{307}$$

The term with no derivatives vanishes in the expansion (307), $v_2(X)v_1(X) - v_1(X)v_2(X) = 0$, which means that the static term in the derivative expansion does not contribute to the source. For a smooth Higgs profile, the derivatives with respect to the time coordinate and $n>1$ are associated with higher powers of v_ω/L_ω, where v_ω and L_ω are the velocity and the width of the bubble wall, respectively. Since the typical time scale of the processes giving rise to the source is given by the thermalization time of the higgsinos $1/\Gamma_{\widetilde{H}}$, the approximation is good for values of $L_\omega\Gamma_{\widetilde{H}}/v_\omega \gg 1$. In other words, this expansion is valid only when the mean free path of the higgsinos in the plasma is smaller than the scale of variation of the Higgs background determined by the wall thickness, L_ω, and the wall velocity v_ω. The term corresponding to $n=1$ in the expansion (307) gives a contribution to the source proportional to the function

$$v_1(X)\partial^\mu_X v_2(X) - v_2(X)\partial^\mu_X v_1(X) \equiv v^2(X)\partial^\mu_X\beta(X), \tag{308}$$

which should vanish smoothly for values of X outside the bubble wall. Here we have denoted $v^2 \equiv v_1^2 + v_2^2$. Since the variation of the Higgs fields is due to the expansion of the bubble wall through the thermal bath, the source $\gamma_{\widetilde{H}}$ will be linear in v_ω. The corresponding contribution tot he CP violating source reads

$$\gamma_{\widetilde{H}}(X) = \mathrm{Im}(\mu)\,\left[v^2(X)\dot{\beta}(X)\right]\left[3M_2\, g_2^2\,\mathcal{I}^{\widetilde{W}}_{\widetilde{H}} + M_1\, g_1^2\,\mathcal{I}^{\widetilde{B}}_{\widetilde{H}}\right], \tag{309}$$

where

$$\mathcal{I}^{\widetilde{W}}_{\widetilde{H}} = \int_0^\infty dk\frac{k^2}{2\pi^2\omega_{\widetilde{H}}\omega_{\widetilde{W}}}$$

$$
\begin{aligned}
\Big[\quad & \left(1-2\mathrm{Re}(f^0_{\widetilde{W}})\right) I(\omega_{\widetilde{H}},\Gamma_{\widetilde{H}},\omega_{\widetilde{W}},\Gamma_{\widetilde{W}}) + \left(1-2\mathrm{Re}(f^0_{\widetilde{H}})\right) I(\omega_{\widetilde{W}},\Gamma_{\widetilde{W}},\omega_{\widetilde{H}},\Gamma_{\widetilde{H}}) \\
+ \quad & 2\left(\mathrm{Im}(f^0_{\widetilde{H}}) + \mathrm{Im}(f^0_{\widetilde{W}})\right) G(\omega_{\widetilde{H}},\Gamma_{\widetilde{H}},\omega_{\widetilde{W}},\Gamma_{\widetilde{W}}) \quad \Big]
\end{aligned}
\tag{310}
$$

and $\omega^2_{\widetilde{H}(\widetilde{W})} = k^2 + |\mu|^2(M_2^2)$ while $f^0_{\widetilde{H}(\widetilde{W})} = 1/\left[\exp\left(\omega_{\widetilde{H}(\widetilde{W})}/T + i\Gamma_{\widetilde{H}(\widetilde{W})}/T\right)+1\right]$. The functions I and G are given by

$$
\begin{aligned}
I(a,b,c,d) &= \frac{1}{2}\frac{1}{[(a+c)^2+(b+d)^2]}\sin\left[2\mathrm{arctan}\frac{a+c}{b+d}\right] \\
&+ \frac{1}{2}\frac{1}{[(a-c)^2+(b+d)^2]}\sin\left[2\mathrm{arctan}\frac{a-c}{b+d}\right], \\
G(a,b,c,d) = &- \frac{1}{2}\frac{1}{[(a+c)^2+(b+d)^2]}\cos\left[2\mathrm{arctan}\frac{a+c}{b+d}\right] \\
&- \frac{1}{2}\frac{1}{[(a-c)^2+(b+d)^2]}\cos\left[2\mathrm{arctan}\frac{a-c}{b+d}\right].
\end{aligned}
\tag{311}
$$

Notice that the function $G(\omega_{\widetilde{H}},\Gamma_{\widetilde{H}},\omega_{\widetilde{W}},\Gamma_{\widetilde{W}})$ has a peak for $\omega_{\widetilde{H}} \sim \omega_{\widetilde{W}}$. This resonant behaviour is associated to the fact that the Higgs background is carrying a very low momentum (of order of the inverse of the bubble wall width L_ω) and to the possibility of absorption or emission of Higgs quanta by the propagating supersymmetric particles. The resonance can only take place when the higgsino and the wino do not differ too much in mass. By using the Uncertainty Principle, it is easy to understand that the width of this resonance is expected to be proportional to the thermalization rate of the particles giving rise to the baryon asymmetry.

The damping rate of charged and neutral Higgsinos is expected to be of the order of $5\times 10^{-2}T$. The Bino contribution may be obtained from the above expressions by replacing M_2 by M_1. The CP violating source for the Higgs fermion number is enhanced if $M_2, M_1 \sim \mu$ and low momentum particles are transmitted over the distance L_ω. This means that the classical approximation is not entirely adequate to describe the quantum interference nature of CP violation and only a quantum approach is suitable for the computation of the building up of the CP violating sources. Notice that the source is built up integrating over all the history of the system. This leads to "memory effect" that are responsible for some enhancement of the final baryon asymmetry. These memory effects lead to "relaxation" times for the CP violating sources which are typically longer than the ones dictated by the thermalization rates of the particles in the thermal bath. In fact, this observation is valid for all the processes described by the "scattering" term in the right-handed side of the quantum diffusion equations. The slowdown of the relaxation processes may help to keep the system out of equilibrium for longer times and therefore enhance the final baryon asymmetry. There are two more reasons why one should expect quantum relaxation times to be longer than the ones predicted by the classical approach. First, the decay of the Green's functions as functions of the difference of the time arguments: an exponential decay is found in thermal equilibrium when one ignore the frequency dependence of self-energies in the spectral functions, *e.g.* $|G^>(\mathbf{k},t,t')| \sim |G^>(\mathbf{k})| \times \exp\left[-\Gamma(\mathbf{k},\omega)|t-t'|\right]$. The decay of the Green's functions restrict the range of the time integration for the scattering term, reduces the integrals and, therefore, the change of the local particle number densities as a function of time. The second effect is the rather different oscillatory behaviour of the functions $G^>$ and $G^<$ for a given momentum, as functions of the time argument difference.

As we have previously mentioned, the final baryon asymmetry (273) depends sensitively on the parameter $\mathcal{A}$. The parameter $\mathcal{A}$ computed from the higgsino source is

$$\begin{aligned}\mathcal{A} &\propto \frac{2f(k_i)\Gamma_{\widetilde{H}}}{\overline{D}\,\lambda_+} I,\\ I &\equiv \int_0^\infty du\, v^2(u)\frac{d\beta(u)}{du} e^{-\lambda_+ u} \simeq \int_0^\infty du\, v^2(u)\frac{d\beta(u)}{du},\end{aligned} \tag{312}$$

where $f(k_i)$ is a coefficient depending upon the number of degrees of freedom present in the thermal bath. The integral I has been computed including two-loop effects in ref. [106] and results to be $I \simeq 10^{-2}$ for $m_A = 150$–200 GeV. The final baryon asymmetry turns out to be [110]

$$\frac{n_B}{s} \simeq \left(\frac{|\sin(\phi_\mu)|}{10^{-3}}\right) 4\times 10^{-11}, \tag{313}$$

for $v_\omega \simeq 1$. It is intriguing that these small values of the phases are perfectly consistent with the constraints from the electric dipole moment of the neutron and squarks of the first and second generation as light as ~ 100 GeV may be tolerated.

8 Conclusions

In these lectures we have learned that cosmology provides really strong arguments in favour of the nonconservation of the baryon number. The SM of weak interactions, which is so successfull in explaining the experimental data obtained at accelerator machines operating at energy scales of about 100 GeV, seems unable to explain the observed baryon asymmetry of the Universe. This is a very strong indication that there is some new, yet undiscovered, physics beyond the SM. We do not know whether this is just the low energy supersymmetric extension of the SM. If so, we can draw tight constraints on the Higgs spectrum of the MSSM and the next generation of accelerator machines, such as LHC, will tell us if this is a tenable option. It might be that this cosmological puzzle has been taken care of by some new physics at energy scales much higher than the weak scale, the GUT scale, as suggested by gauge coupling unification. Even though this option is not testable at particle colliders, the most striking evidence of baryon number violation might come from the detection of proton decay. It is very exciting that in the next few years we will be able to confirm (or disprove) some of the theories of baryogenesis.

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Solution to the Exercises

1) Suppose that the baryon number B is conserved by the interactions. This means that the baryon number commutes with the hamiltonian of the system H, $[B, H] = 0$. Therefore, supposing that $B(t_0)$=0, we have $B(t) \propto \int_{t_0}^{t} [B, H] dt' = 0$ at all times and no baryon number production may take place.

2) Let us consider a model universe with two components: inflaton field energy, ρ_ϕ and radiation energy density, ρ_R. We will assume that the decay rate of the inflaton field energy density is Γ_ϕ. We will also assume that the light degrees of freedom are in local thermodynamic equilibrium.

With the above assumptions, the Boltzmann equations describing the redshift and interchange in the energy density among the different components is

$$\begin{aligned} &\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0 \\ &\dot{\rho}_R + 4H\rho_R - \Gamma_\phi \rho_\phi = 0, \end{aligned} \tag{314}$$

where dot denotes time derivative.

It is useful to introduce the dimensionless constant, α_Φ defined in terms of Γ_ϕ as

$$\Gamma_\phi = \alpha_\phi M_\phi. \tag{315}$$

For a reheat temperature much smaller than M_ϕ, Γ_ϕ must be small.

It is also convenient to work with dimensionless quantities that can absorb the effect of expansion of the universe. This may be accomplished with the definitions

$$\Phi \equiv \rho_\phi M_\phi^{-1} a^3 \ ; \quad R \equiv \rho_R a^4. \tag{316}$$

It is also convenient to use the scale factor, rather than time, for the independent variable, so we define a variable $x = aM_\phi$. With this choice the system of equations can be written as (prime denotes d/dx)

$$\begin{aligned} \Phi' &= -c_1 \frac{x}{\sqrt{\Phi x + R}} \Phi \\ R' &= c_1 \frac{x^2}{\sqrt{\Phi x + R}} \Phi. \end{aligned} \tag{317}$$

The constant c_1 is given by

$$c_1 = \sqrt{\frac{3}{8\pi}} \frac{M_{\rm Pl}}{M_\phi} \alpha_\phi. \tag{318}$$

It is straightforward to solve the system of equations in Eq. (317) with initial conditions at $x = x_I$ of $R(x_I) = 0$ and $\Phi(x_I) = \Phi_I$. It is convenient to express $\rho_\phi(x = x_I)$ in terms of the expansion rate at x_I, which leads to

$$\Phi_I = \frac{3}{8\pi}\frac{M_{\rm P}^2}{M_\phi^2}\frac{H_I^2}{M_\phi^2}x_I^3. \tag{319}$$

The numerical value of x_I is irrelevant.

Before solving the system of equations, it is useful to consider the early-time solution for R. Here, by early time, we mean $H \gg \Gamma_\Phi$, *i.e.*, before a significant fraction of the comoving coherent energy density is converted to radiation. At early times $\Phi \simeq \Phi_I$, and $R \simeq X \simeq 0$, so the equation for R' becomes $R' = c_1 x^{3/2}\Phi_I^{1/2}$. Thus, the early time solution for R is simple to obtain:

$$R \simeq \frac{2}{5}c_1\left(x^{5/2} - x_I^{5/2}\right)\Phi_I^{1/2} \qquad (H \gg \Gamma_\phi)\ . \tag{320}$$

Now we may express T in terms of R to yield the early-time solution for T:

$$\frac{T}{M_\phi} \simeq \left(\frac{12}{\pi^2 g_*}\right)^{1/4} c_1^{1/4}\left(\frac{\Phi_I}{x_I^3}\right)^{1/8}\left[\left(\frac{x}{x_I}\right)^{-3/2} - \left(\frac{x}{x_I}\right)^{-4}\right]^{1/4} \qquad (H \gg \Gamma_\Phi)\ . \tag{321}$$

Thus, T has a maximum value of

$$\begin{aligned}\frac{T_{MAX}}{M_\phi} &= 0.77\left(\frac{12}{\pi^2 g_*}\right)^{1/4} c_1^{1/4}\left(\frac{\Phi_I}{x_I^3}\right)^{1/8} \\ &= 0.77\alpha_\phi^{1/4}\left(\frac{9}{2\pi^3 g_*}\right)^{1/4}\left(\frac{M_{\rm P}^2 H_I}{M_\phi^3}\right)^{1/4}\ ,\end{aligned} \tag{322}$$

which is obtained at $x/x_I = (8/3)^{2/5} = 1.48$. It is also possible to express α_ϕ in terms of T_{RH} and obtain

$$\frac{T_{MAX}}{T_{RH}} = 0.77\left(\frac{9}{2\pi^3 g_*}\right)^{1/4}\left(\frac{H_I M_{\rm P}}{T_{RH}^2}\right)^{1/4}\ . \tag{323}$$

For an illustration, in the simplest model of chaotic inflation $H_I^2 \sim M_{\rm P} M_\phi$ with $M_\phi \simeq 10^{13}$GeV, which leads to $T_{MAX}/T_{RH} \sim 2\times 10^3 (200/g_*)^{1/4}$ for $T_{RH} = 10^9$GeV.

We can see from Eq. (320) that for $x/x_I > 1$, in the early-time regime T scales as $a^{-3/8}$. So entropy is created in the early-time regime. So if one is producing a massive particle during reheating it is necessary to take into account the fact that the maximum temperature is greater than T_{RH}, and during the early-time evolution $T \propto a^{-3/8}$.

3) The euclidean action is given by

$$S_3 = 4\pi\int_0^R r^2\, dr\left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi)\right], \tag{324}$$

where R is the radius of the bubble. If we now indicate by δR the thickness of the bubble wall and $\Delta V = V(\phi_2) - V(\phi_1) < 0$, we get

$$S_3 \sim 2\pi R^2 \delta R\left(\frac{\delta\phi}{\delta R}\right)^2 \delta R + \frac{4\pi R^3 \Delta V}{3}, \tag{325}$$

where $\delta\phi = \phi_2 - \phi_1$. Suppose now that the bubbles are thick, that is $\delta R \sim R$. In such a case

$$S_3 \sim 2\pi R(\delta\phi)^2 + \frac{4\pi R^3 \Delta V}{3}. \tag{326}$$

The critical radius R_c is obtained as the maximum of the action (326)

$$R_c \sim \frac{\delta\phi}{\sqrt{-2\Delta V}}. \tag{327}$$

4) We introduce a chemical potential for any particle which takes part to fast processes, and then reduce the number of linearly independent chemical potentials by solving the corresponding system of equations. Finally, we can express the abundances of any particle in equilibrium in terms of the remaining linear independent chemical potentials, corresponding to the conserved charges of the system.

Since strong interactions are in equilibrium inside the bubble wall, we can chose the same chemical potential for quarks of the same flavour but different color, and set to zero the chemical potential for gluons. Moreover, since inside the bubble wall $SU(2)_L \times U(1)_Y$ is broken, the chemical potential for the neutral Higgs scalars vanishes[4].

The other fast processes, and the corresponding chemical potential equations are:

i) top Yukawa:

$$\begin{array}{lll} t_L + H_2^0 \leftrightarrow t_R + g, & (\mu_{t_L} = & \mu_{t_R}), \\ b_L + H^+ \leftrightarrow t_R + g, & (\mu_{t_R} = & \mu_{b_L} + \mu_{H^+}), \end{array} \tag{328}$$

ii) $SU(2)_L$ flavour diagonal:

$$\begin{array}{lll} e_L^i \leftrightarrow \nu_L^i + W^-, & (\mu_{\nu_L^i} = & \mu_{e_L^i} + \mu_{W^+}), \\ u_L^i \leftrightarrow d_L^i + W^+, & (\mu_{u_L^i} = & \mu_{d_L^i} + \mu_{W^+}), \\ H_2^0 \leftrightarrow H^+ + W^-, & (\mu_{H^+} = & \mu_{W^+}), \\ H_1^0 \leftrightarrow H^- + W^+, & (\mu_{H^-} = & -\mu_{W^+}), \end{array} \quad (i = 1,\ 2,\ 3). \tag{329}$$

Neutral current gauge interactions are also in equilibrium, so we have zero chemical potential for the photon and the Z boson.

Imposing the above constraints, we can reduce the number of independent chemical potentials to four, μ_{W^+}, μ_{t_L}, $\mu_{u_L} \equiv 1/2 \sum_{i=1}^{2} \mu_{u_L^i}$, and $\mu_{e_L} \equiv 1/3 \sum_{i=1}^{3} \mu_{e_L^i}$. These quantities correspond to the four linearly independent conserved charges of the system. Choosing the basis Q, $(B-L)$, $(B+L)$, and $BP \equiv B_3 - 1/2(B_1 + B_2)$, where the primes indicate that only particles in equilibrium contribute to the various charges, and introducing the respective chemical potentials, we can go to the new basis using the relations

$$\left\{ \begin{array}{lll} \mu_Q & = & 3\mu_{t_L} + 2\mu_{u_L} - 3\mu_{e_L} + 11\mu_{W^+}, \\ \mu_{(B-L)} & = & 3\mu_{t_L} + 4\mu_{u_L} - 6\mu_{e_L} - 6\mu_{W^+}, \\ \mu_{(B+L)} & = & 3\mu_{t_L} + 4\mu_{u_L} + 6\mu_{e_L}, \\ \mu_{BP} & = & 3\mu_{t_L} - 2\mu_{u_L}. \end{array} \right. \tag{330}$$

[4] This is true if chirality flip interactions, or processes like $Z \to Z^* h$, are sufficiently fast.

If sphaleron transitions were fast, then we could eliminate a further chemical potential through the constraint

$$3\sum_{i=1}^{3}\mu_{u^i_L}+3\sum_{i=1}^{3}\mu_{d^i_L}+\sum_{i=1}^{3}\mu_{e^i_L}=0. \tag{331}$$

In this case, the value of $(B+L)$ would be determined by that of the other three charges according to the relation

$$(B+L)_{\rm EQ}=\frac{3}{80}Q+\frac{7}{20}BP-\frac{19}{40}(B-L). \tag{332}$$

The above result should not come as a surprise, since we already know that a non zero value for $B-L$ gives rise to a non zero $(B+L)$ at equilibrium. Stated in other words, sphaleron transitions erase the baryon asymmetry only if any conserved charge of the system has vanishing thermal average, otherwise the equilibrium point lies at $(B+L)_{\rm EQ}\neq 0$.

At high temperature ($\mu_i \ll T$) the free energy of the system is given by

$$\begin{aligned}F=\tfrac{T^2}{12}\quad&\Big[3\mu^2_{e_L}+3\mu^2_{\nu_L}+6\mu^2_{u_L}+3\mu^2_{t_L}+3\mu^2_{t_R}+6\mu^2_{d_L}+3\mu^2_{b_L}\\&+6\mu^2_{W^+}+2\mu^2_{H^+}+2\mu^2_{H^0_1}+2\mu^2_{H^0_2}\Big].\end{aligned} \tag{333}$$

Using (328), (329) and (330) to express the chemical potentials in terms of the four conserved charges in (330) we obtain the free energy as a function of the density of $(B+L)=\mu_{B+L}T^2/6$,

$$F\left[(B+L)\right]=0.46\frac{\left[(B+L)-(B+L)_{\rm EQ}\right]^2}{T^2}+\quad\text{constant terms}, \tag{334}$$

where the *"constant terms"* depend on Q, $(B-L)$, and BP but not on $(B+L)$, and $(B+L)_{\rm EQ}$ is given by (332).

From the above expression, we can see that

$$\frac{dn_{B+L}}{dt}\propto-\frac{\Gamma_{\rm sp}}{T}\frac{\partial F}{\partial(B+L)}\propto-\frac{\Gamma_{\rm sp}}{T}\left[(B+L)-(B+L)_{\rm EQ}\right]. \tag{335}$$

5) The contribution to the E parameter of the potential (231) is given generically by, see Eq. (229),

$$-\frac{T}{12\pi}\sum_i n_i\left[m_i^2(\phi)+\Pi_i(T)\right]^{3/2}, \tag{336}$$

for a generic bosonic particle i with plasma mass $\Pi_i(T)$. In the case of the right-handed stop, we get

$$\delta E=-2\,N_c\,\frac{T}{12\pi}\left[m^2_{\widetilde{t}}+\Pi_R(T)\right]^{3/2}, \tag{337}$$

where $N_c=3$ is the number of color and $m^2_{\widetilde{t}}$ is given in (251). The upper bound on the contribution to the E parameter from the right-handed stops is obtained when $m_U^2<0$ and $m^{\rm eff}_{\widetilde{t}}=m_U^2+\Pi_R(T)\simeq 0$. This gives

$$\delta E\lesssim\frac{h_t^3}{2\pi}\left(1-\widetilde{A}_t^2/m_Q^2\right)^{3/2}. \tag{338}$$

Using now the fact that $m_t = h_t v$ and that $\langle\phi(T_c)\rangle/T_c = 2E/\lambda \simeq 4v^2E/m_h^2$, we get Eq. (253).